Vol. 82, No. 2 DUKE MATHEMATICAL JOURNAL © February 1996

MORE IRREDUCIBLE BOUNDARY REPRESENTATIONS OF FREE GROUPS

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0. Introduction. Let Γ be a noncommutative free group on finitely many generators. Fix a basis for Γ and let A consist of the basis elements and their inverses. The *Cayley graph* of Γ with respect to A, denoted by $\mathscr{T}$, has Γ as its vertex set and has an edge between each pair of vertices $\{x, xa\}$ for $x \in \Gamma$ and $a \in A$. The left action of Γ on itself clearly preserves the graph structure. It is well known that $\mathscr{T}$ is an infinite tree and is homogeneous, meaning that each vertex lies on the same number of edges.

A *geodesic* in $\mathscr{T}$ is a sequence of vertices, $(x_j)_{j=1}^J$, so that for all $j \leqslant J-1$, x_j and x_{j+1} are joined by an edge and so that for no $j \leqslant J-2$ does $x_j = x_{j+2}$. We admit the possibility $J = \infty$, that is, the possibility of semi-infinite geodesics. Between each pair of vertices $(x, y) \in \Gamma$, there is a unique geodesic, denoted by $[x, y]$. The *boundary*, Ω, of $\mathscr{T}$, is the set of all semi-infinite geodesics in $\mathscr{T}$, modulo the following equivalence relation:

$$(x_j)_{j=0}^\infty \sim (y_j)_{j=0}^\infty \quad \text{if there exist } j_1 \text{ and } j_2 \text{ so that } x_{j_1+j} = y_{j_2+j} \quad \text{for all } j \geqslant 0.$$

One imagines a point $\omega \in \Omega$ as being the limit of the vertices of any geodesic representing it. The left action of Γ on $\mathscr{T}$ extends in an obvious way to an action on Ω.

It is easy to see, using the properties of trees, that each $\omega \in \Omega$ has a unique representing geodesic which starts at $e \in \Gamma$. Denote this geodesic by $[e, \omega] = (\omega_j)_{j=0}^\infty$. For $x \in \Gamma$, denote by $(x_j)_{j=0}^J$ the geodesic $[e, x]$, and extend this to $(x_j)_{j=0}^\infty$ by setting $x_j = \varnothing$ for $j > J$. Let $|x|$ denote the number of edges in $[e, x]$, namely J if x is as above. Alternatively, $|x|$ is the number of letters in the *reduced word* for x, that is, in the unique shortest expression of x as a product of elements of A. For $z \in \Gamma \cup \Omega$,

$$(z_j)_{j=0}^\infty \in \prod_{j=0}^\infty (\{y; |y| = j\} \cup \{\varnothing\}).$$

Use the product topology for the right-hand side and use the subspace topology for $\Gamma \cup \Omega$. The subspace $\{(z_j)_{j=0}^\infty; z \in \Gamma \cup \Omega\}$ is closed, therefore compact, so $\Gamma \cup \Omega$

Received 27 July 1994. Revision received 5 May 1995.

Kuhn thanks the University of Chicago for its generous hospitality.

Steger partially supported by the National Science Foundation. His visit to Italy in the summer of 1991 was financed by the Consiglio Nazionale delle Ricerche.

The material in the appendix to earlier versions has been repositioned as Section 2.