Vol. 82, No. 2 DUKE MATHEMATICAL JOURNAL © February 1996

TOPOLOGICAL QUANTUM FIELD THEORIES FOR SURFACES WITH SPIN STRUCTURE

C. BLANCHET AND G. MASBAUM

Introduction. A *Topological Quantum Field Theory* (TQFT) in dimension 3 is a functor from a $2 + 1$-dimensional cobordism category to a category of modules, satisfying certain axioms. This terminology was introduced by Atiyah [1] following Witten's interpretation [32], in terms of quantum field theory, of the Jones-polynomial invariant of links in the 3-sphere. The TQFT-axioms imply that the functor is determined by its values on closed bordisms. These lie in the ground ring, and are 3-manifold invariants. They are sometimes called *quantum invariants.* Existence of quantum invariants was first proven by Reshetikhin and Turaev [28]. Other constructions of invariants were given, e.g., in [6], [16], [17], [18], [19], [27], [31]. A construction of TQFT-functors based on the invariants of [6] was given in [8].

Refined quantum invariants which depend nontrivially on a choice of spin structure on the manifold were constructed independently by Kirby and Melvin [16], Turaev [29] (using quantum groups), and the first author [5] (using the Kauffman bracket [13]). As a special case, these invariants include (a version of) the well-known classical *Rohlin* (or *μ-*)*-invariant.*

A formula of [5], [16], [29] asserts that the sum of the spin invariants of a closed 3-manifold is equal to the "unspun" invariant. The question arises of how this generalizes on the level of TQFTs. A partial answer was given already in [8], where it was shown that the V_{8k}-module of a surface is naturally decomposed into a direct sum of submodules associated to spin structures on the surface.

The philosophy of [8] was to start from the 3-manifold invariant, extend it by a universal construction to a functor on a cobordism category, and then prove the TQFT-axioms. In the present paper, we use the same philosophy but start from the spin-refined invariants. The aim is to find out what a "Spin TQFT" should be, and to understand its relationship with the "unspun" theory.

We consider a series of functors V^s_{8k} on $2 + 1$-dimensional spin cobordism categories $\mathscr{C}^s_{8k}$. (The indexing by $8k$, $k \geqslant 1$, is for notational coherence with [8].) These functors are constructed from (a suitable renormalization of) the spin invariants of [5]. They are quantization functors (in the sense of [8]), and satisfy surgery properties and (spin) Kauffman relations at a primitive $16k$th root of unity.

There are three main results. The first is that the V^s_{8k}-modules associated to surfaces are free of finite rank (Theorem 7.3). We also give, in the last section,

Received 5 August 1994. Revision received 13 June 1995.