Vol. 82, No. 1 DUKE MATHEMATICAL JOURNAL January 1996

ADDENDUM TO "BROWNIAN MOTION AND THE FUNDAMENTAL FREQUENCY OF A DRUM"

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We began the paper [1] by quoting the following theorem.

THEOREM. *Let D be a simply connected domain in the complex plane. Let R_D be the inradius of D, that is, the radius of the largest disc contained in D, and let λ_D be the first Dirichlet eigenvalue for the Laplacian in D. Then there is a universal constant a such that*

$$(*) \qquad \lambda_D \geq \frac{a}{R_D^2}.$$

We, as well as many other people who worked on this and related results (see the extensive literature cited in [1]), attributed the above theorem to W. K. Hayman [3]. We have recently learned from Mark Ashbaugh (via Richard Laugesen) that this result was first proved in 1965 by the now-deceased Hungarian mathematician Endre Makai [4]. Mark Ashbaugh has also informed us that he learned about Makai's result from the paper of Gabriella Bognár [2], which deals with a similar result for the p-Laplacian. It is perhaps also interesting to note that Bognár in her paper thanks Endre Makai Jr, the son of Makai and himself a mathematician, for "stimulating conversations" and for giving her some references. Finally, Makai's proof also shows that the best a in $(*)$ satisfies $1/4 \leq a < \pi^2/4$, which was the best estimate prior to the results in [1].

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Received 23 October 1995.