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MULTIPLICITIES FORMULA FOR GEOMETRIC QUANTIZATION, PART II

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1. Introduction. Let P be a compact manifold. Let H be a compact Lie group acting on the right on P. We assume that the stabilizer of each element $y \in P$ is a finite subgroup of H. The space $M = P/H$ is an orbifold and every orbifold can be presented this way. If H acts freely, then M is a manifold. If $\mathscr{L}$ is an H-equivariant line bundle on P, the space $\mathscr{L}/H$ will be called an *orbifold line bundle* on M. Let G be a compact Lie group with Lie algebra $\mathfrak{g}$ acting on the compact orbifold $M = P/H$. We consider the case where M is a prequantized symplectic orbifold. Let $\mathscr{L}$ be a G-equivariant Kostant-Souriau orbifold line bundle on M. Then the quantized representation $Q(M, \mathscr{L})$ associated to $(M, \mathscr{L})$ is a virtual representation of G constructed as the $\mathbb{Z}/2\mathbb{Z}$-graded space of H-invariant solutions of the H-horizontal Dirac operator on P twisted by the line bundle $\mathscr{L}$.

Let $\mu: M \to \mathfrak{g}^*$ be the moment map for the G-action. Assume 0 is a regular value of μ. Let M_{red} be the reduced orbifold of M; that is, $M_{\text{red}} = \mu^{-1}(0)/G$. Consider the reduced orbifold line bundle $\mathscr{L}_{\text{red}} = \mathscr{L}|_{\mu^{-1}(0)}/G$ on M_{red}. In the case where both G and H are torus, we prove here the formula

$$Q(M, \mathscr{L})^G = Q(M_{\text{red}}, \mathscr{L}_{\text{red}}).$$

This formula was conjectured by Guillemin-Sternberg [4] and proved when M is a complex manifold and $\mathscr{L}$ a sufficiently positive G-equivariant holomorphic line bundle. Here, we do not assume the existence of complex structure on M. Initially, we obtained a proof [7] of the formula $Q(M, \mathscr{L})^G = Q(M_{\text{red}}, \mathscr{L}_{\text{red}})$ for the case where M is a symplectic manifold with Hamiltonian action of a torus G such that G acts freely on $\mu^{-1}(0)$. Let us recall that independently E. Meinrenken [6] had obtained a proof of the formula $Q(M, \mathscr{L})^G = Q(M_{\text{red}}, \mathscr{L}_{\text{red}})$ including the case where M_{red} is an orbifold. It is possible to generalise the method sketched in [7] to cover the case of orbifolds. Indeed, after writing a character formula [9] for $Q(P/H, \mathscr{L})$, similar arguments can be given. We give here an alternative approach that requires almost no calculations. This approach is the K-theoretical version of the deformation argument in equivariant cohomology employed in Part I of this article [8]. However, we have tried to write the present article in such a way that the reading of Part I (although reassuring) is not necessary to understand our arguments. In Part I, we wrote in detail the case of an S^1-action using a deformation formula for the character of $Q(M, \mathscr{L})$. The original inspiration of

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