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OSCILLATORY INTEGRALS AND MAXIMAL AVERAGES OVER HOMOGENEOUS SURFACES

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1. Introduction. Let S be a smooth hypersurface in R^{n+1}, let $d\sigma$ denote Lebesgue measure on S, and let ψ denote a smooth cutoff function in R^{n+1}. Let δ_t denote the dilation $\delta_t h(x, x_{n+1}) = t^{-n} h(t^{-1}x, t^{-1}x_{n+1})$. We consider the convolution operators

$$M_t f(x, x_{n+1}) = f * \delta_t(\psi d\sigma)(x, x_{n+1})$$

and their associated maximal operator

$$\mathscr{M}f(x, x_{n+1}) = \sup_{t>0} M_t f(x, x_{n+1}). \tag{1}$$

It is not obvious that such convolutions are well defined for f in L^p spaces since S has measure zero in R^{n+1}. Nevertheless, a priori L^p estimates are possible when S has suitable curvature properties. A basic problem is thus to determine the optimal range of indices p such that

$$\|\mathscr{M}f\|_{L^p(R^{n+1})} \leqslant C_p \|f\|_{L^p(R^{n+1})}, \tag{2}$$

where f is initially taken to be in the class of rapidly decreasing functions.

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