Vol. 82, No. 1 DUKE MATHEMATICAL JOURNAL © January 1996

MODULAR INVARIANT THEORY OF PARABOLIC SUBGROUPS OF $GL_n(\mathbb{F}_q)$ AND THE ASSOCIATED STEENROD MODULES

T. J. HEWETT

§1. Introduction. The invariant theory of $GL_n(\mathbb{F}_q)$ was described by a theorem of Dickson as follows.

THEOREM 1.1 (Dickson). *If $GL_n(\mathbb{F}_q)$ acts canonically on the polynomial algebra $\mathbb{F}_q[x_1, x_2, \ldots, x_n]$, then the invariant subalgebra is polynomial on generators $D_{n,1}$, $D_{n,2}, \ldots, D_{n,n}$, given by the coefficients of the polynomial $f_n(x)$.*

$$f_n(x) = \prod(x - \alpha_1 x_1 - \alpha_2 x_2 - \cdots - \alpha_n x_n)$$

$$= x^{q^n} - D_{n,1}x^{q^{n-1}} + D_{n,2}x^{q^{n-2}} - \cdots (-1)^n D_{n,n}x,$$

where $(\alpha_1, \alpha_2 \cdots \alpha_n)$ run over all of $\mathbb{F}_q^n$. One also has

$$D_{n,i} = \frac{\operatorname{Det}\begin{bmatrix} x_1^{q^n} & x_2^{q^n} & \cdots & x_n^{q^n} \\ \cdot & \cdot & & \cdot \\ \widehat{x_1^{q^{n-i}}} & \widehat{x_2^{q^{n-i}}} & \cdots & \widehat{x_n^{q^{n-i}}} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ x_1 & x_2 & \cdots & x_n \end{bmatrix}}{\operatorname{Det}\begin{bmatrix} x_1^{q^{n-1}} & x_2^{q^{n-1}} & \cdots & x_n^{q^{n-1}} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ x_1 & x_2 & \cdots & x_n \end{bmatrix}}$$

so $\operatorname{Deg}(D_{n,i}) = q^{n-i}(q^i - 1)$.

We will show here that this generalizes to give the modular invariants of any

Received 12 December 1994. Revision received 16 May 1995.