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PRODUCT EXPANSIONS FOR ZETA FUNCTIONS ATTACHED TO LOCALLY SYMMETRIC SPACES OF HIGHER RANK

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0. Introduction. In 1976 D. Ruelle showed in a remarkable paper [Rue] that the dynamical zeta function of a hyperbolic flow with analytic foliations admits an analytic continuation to the entire complex plane. Moreover, it is a quotient of Fredholm determinants of suitable transfer operators. These, however, depend on a local construction on the manifald without reasonable invariance properties. One wants to replace this construction by a different approach, which bears more global information of the manifold encoded in the analysis of zeta functions. Up to now, this only works well in special cases as locally symmetric spaces of rank one, where one can use harmonic analysis of the underlying isometry group. Here several authors investigated the analysis of the corresponding zeta functions [Ju], [Fr1], [Wak]. There is a remarkable connection to topological invariants as Reidemeister torsion [Fr1].

Seemingly completely out of range for a general approach is the theory of dynamics with higher dimensional time, i.e., where the action of the one-dimensional time flow is replaced by an action of some finite-dimensional real vector space, with an open cone giving the positive time direction. The sphere bundle of a locally symmetric space of higher rank has a stratification according to different time dimensions. There is a generic open stratum where the time dimension equals the rank of the space, and in its closure we find strata with lower time dimensions, down to the stratum of lowest time dimension. The latter is the most accessible one, and it seems that it already contains all global topological information. In the case that the lowest time dimension is one, Moscovici and Stanton [MS] defined to each flat vector bundle φ over X a zeta function $Z_\varphi(s)$, which satisfies a functional equation as s is replaced by $-s$, and in the center of the functional equation, it has the following behavior. If φ is acyclic, we have

$$Z_\varphi(0) = \tau_\varphi^2,$$

where τ_φ is the analytic torsion of φ in the sense of Ray and Singer [RS]; if φ is not acyclic, we have

$$\operatorname{ord}_{s=0} Z_\varphi(s) = \chi_1(\varphi),$$

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