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INTRINSIC HEIGHTS OF STABLE VARIETIES AND ABELIAN VARIETIES

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1. Introduction

1.1. Heights of polarized varieties. For any $(d, \delta) \in \mathbb{N}^* \times \mathbb{N}^*$, let $\mathscr{V}(d, \delta)$ be the set of isomorphism classes of pairs (X, L) where X is a projective variety of dimension d over $\overline{\mathbb{Q}}$ and L is an ample line bundle over X of degree δ. In various diophantine problems, it is important to dispose of a *height function* on some subset $\mathscr{S}$ of $\mathscr{V}(d, \delta)$, i.e., of a map

$$h: \mathscr{S} \to \mathbb{R},$$

which is bounded from below and such that, for any real numbers D and M, the set of elements x in $\mathscr{S}$ satisfying the conditions:

(1.1.1)
x is the class of a pair (X, L) defined over a number field of degree at most D

and

$$h(x) \leqslant M \tag{1.1.2}$$

is finite.

Clearly, for trivial reasons, there are plenty of (useless) height functions on $\mathscr{V}(d, \delta)$. To be useful, any definition should be as "natural and intrinsic" as possible. Namely, the height of a pair (X, L) should be defined, and reasonably computable, in terms of the intrinsic geometry of (X, L) and of its models over rings of integers of number fields.

The archetype of such natural intrinsic heights is Faltings height of abelian varieties. Let us briefly recall its definition [Fa1]. Let A be an abelian variety of dimension g over $\overline{\mathbb{Q}}$. There exists some number field K on which A may be defined and admits semistable reduction. Let $\pi: \mathscr{A} \to S := \operatorname{Spec} \mathcal{O}_K$ be a semi-abelian model of A, $\varepsilon: S \to \mathscr{A}$ its zero-section, and consider the line bundle on S

$$\omega_{\mathscr{A}/S} := \varepsilon^* \Omega^g_{\mathscr{A}/S} \simeq \pi_* \Omega^g_{\mathscr{A}/S}. \tag{1.1.3}$$

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