THE NEUMANN PROBLEM FOR ELLIPTIC EQUATIONS WITH NONSMOOTH COEFFICIENTS: PART II

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1. Introduction. Let $L = \mathrm{div} A \nabla$ be a real, symmetric, second-order elliptic operator defined in the unit ball of $\mathbb{R}^n$, whose coefficients are assumed to be merely bounded and measurable. That is, if $A = (a_{ij}(X))$, then $a_{ij} \in L^\infty$ and there exists a λ, called the ellipticity constant of L, such that for all $\xi \in \mathbb{R}^n$, $a_{ij}(X)\xi_i\xi_j > \lambda^{-1}|\xi|^2$. In [KP], the systematic study of the Neumann and regularity problems was initiated for such operators L, where boundary values are taken in some $L^p(d\sigma)$ space and appropriate nontangential estimates are required on the gradient of the solution. We provided there a definition of weak solution, constructed the Neumann function $N(X, Y)$ (analogous to the Green function), and then formulated the boundary Neumann and regularity problems. The principal object of [KP] was to solve these problems (in the sense of nontangential estimates) for the class of operators L with radially independent coefficients, and then to obtain solvability for operators whose coefficients are small perturbations of the radially independent ones. Here, a small perturbation is understood in the sense of [D], and this definition will be recalled in Section 1.

This paper is a sequel to [KP]. We shall obtain here solvability of Neumann (N) and regularity (R) boundary value problems for operators which are small perturbations of any operator in the class which is known to have solvable (N) and (R) problems in some $L^p(d\sigma)$. Thus, a perturbation theory is obtained in the general sense of [D] like that for the Dirichlet problem: the restriction of perturbing from a radially independent operator is removed. In addition, we consider "large" perturbations of operators in the sense of [FKP]. Here it is shown that the analog of the large perturbation results of [FKP] (proved there for the Dirichlet problem (D)) are in fact valid for the regularity problem (R). Of course, several of the techniques of the proofs herein rely on the ideas in [DK], [D], [KP], and [FKP].

Finally, in Section 3, we consider the relationships that exist between the various boundary value problems (N), (R), and (D) for a specific operator L. Examples arising from the class of operators obtained from the Laplacian in $\mathbb{R}^2_+$ by means of a quasiconformal change of variables seem to indicate that there might be an equivalence between, for example, solvability of (N) in $L^p(d\sigma)$ and solvability of (D) in $L^{p'}(d\sigma)$, for dual exponents p and p'. We will see that most

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