

# A CHARACTERISATION OF THE TIGHT THREE-SPHERE

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1. Introduction	159
1.1. Statement of the results, concepts from contact geometry	
1.2. The tight three-sphere	
1.3. Sketch of the proof	
1.4. Consequences for the dynamics of the Reeb vector field	
2. Boundary estimates of pseudoholomorphic disks	172
3. The singular characteristic foliation	181
4. Filling $F$ by disks, the maximal Bishop family $\mathcal{B}$	183
5. The finite energy plane asymptotic to $P_0$	195
6. An open-book decomposition of $M$	202
7. Tightness, end of the proof of Theorem 1.4	211
8. Detailed bubbling-off, proof of Theorem 1.5	214
References	225

## 1. Introduction

*1.1. Statement of the results, concepts from contact geometry.* In the following,  $M$  is a three-dimensional compact manifold. A contact structure on  $M$  is a plane field distribution  $\xi \rightarrow M$  defined locally by an equation  $\lambda = 0$ . Here  $\lambda$  is a local one-form on  $M$  such that  $\lambda \wedge d\lambda$  is nowhere vanishing. Clearly,  $\lambda$  and  $f\lambda$  determine the same plane field if the function  $f$  does not vanish. Since  $\lambda \wedge d\lambda$  and  $(f\lambda) \wedge d(f\lambda)$  define the same orientation, a contact structure defines a “natural” orientation on the manifold. If the three-manifold is already oriented, we call  $(M, \xi)$  a positive contact manifold in case these two orientations coincide; otherwise, it is a negative contact manifold. A contact structure is co-orientable, if it is defined globally by  $\lambda = 0$ , with a one-form  $\lambda$  on all of  $M$  having the property that  $\lambda \wedge d\lambda$  is a volume form on  $M$ . Such a one-form is a contact form. The set of all contact forms  $\lambda$  satisfying  $\text{kern}(\lambda) = \xi$  consists of two components; the choice of one of them is a co-orientation. A co-orientation can equivalently be defined by an orientation of the normal bundle  $TM/\xi \rightarrow M$  of the contact structure  $\xi$ . Every compact orientable three-manifold admits a contact form (see Martinet [28]).

The standard example of a positive co-oriented contact structure on the three-sphere is the following. View  $S^3$  as the boundary of the one-ball in  $\mathbb{C}^2$  equipped with the orientation induced from the complex orientation on  $\mathbb{C}^2$ . Let  $\xi \subset TS^3$  be the subbundle whose fibre is the maximal complex subspace of  $TS^3$ . Denote by  $\omega$

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