A CHARACTERISATION OF THE TIGHT THREE-SPHERE

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1. Introduction

1.1. Statement of the results, concepts from contact geometry. In the following, M is a three-dimensional compact manifold. A contact structure on M is a plane field distribution $\xi \to M$ defined locally by an equation $\lambda = 0$. Here λ is a local one-form on M such that $\lambda \wedge d\lambda$ is nowhere vanishing. Clearly, λ and $f\lambda$ determine the same plane field if the function f does not vanish. Since $\lambda \wedge d\lambda$ and $(f\lambda) \wedge d(f\lambda)$ define the same orientation, a contact structure defines a "natural" orientation on the manifold. If the three-manifold is already oriented, we call (M, ξ) a positive contact manifold in case these two orientations coincide; otherwise, it is a negative contact manifold. A contact structure is co-orientable, if it is defined globally by $\lambda = 0$, with a one-form λ on all of M having the property that $\lambda \wedge d\lambda$ is a volume form on M. Such a one-form is a contact form. The set of all contact forms λ satisfying kern $(\lambda) = \xi$ consists of two components; the choice of one of them is a co-orientation. A co-orientation can equivalently be defined by an orientation of the normal bundle $TM/\xi \to M$ of the contact structure ξ. Every compact orientable three-manifold admits a contact form (see Martinet [28]).

The standard example of a positive co-oriented contact structure on the three-sphere is the following. View S^3 as the boundary of the one-ball in $\mathbb{C}^2$ equipped with the orientation induced from the complex orientation on $\mathbb{C}^2$. Let $\xi \subset TS^3$ be the subbundle whose fibre is the maximal complex subspace of TS^3. Denote by ω

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