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INTEGRAL HODGE THEORY AND CONGRUENCES BETWEEN MODULAR FORMS

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Introduction. In this work we offer a new perspective on the construction of fusion modules between new and old forms. Our construction is uniformly applicable for all weights $\geqslant 2$. Modules of fusion were introduced by Mazur and studied by various people [18], [29], [30], [8], [23], [9]: see Definition 3.21 for a slight generalization. These modules detect congruences between the Fourier expansions of two normalized newforms. They have several important applications. One is Ribet's work on Serre's "epsilon" conjecture for weight-2 forms with trivial character. In it he completed the Frey-Serre program, showing that the Taniyama-Weil conjecture implies Fermat's Last Theorem [30]. Another is Taylor's construction of Galois representations attached to Hilbert modular forms [37].

The constructions of modules of fusion require *integral models* for spaces of modular forms. One way to get such modules is through equivariant chains (with local coefficients) of (S-)arithmetic groups acting on symmetric spaces [18], [29], [8] or on their p-adic analogs [23], [30]. The modules of fusion usually arise as discriminant forms of Petersson inner products. In this article we propose a somewhat different approach based on Eckmann-Hodge theory [11].

This theory of Eckmann and Hodge shows that cohomology classes of simplicial complexes have unique harmonic representatives when the coefficients are a field of characteristic zero. We get our modules of fusion as an obstruction to this when the coefficients are S-integers. We take $S = \{p\}$, where p is a prime which splits B, and we consider (S-)arithmetic subgroups of the multiplicative group of a definite rational quaternion algebra B. This gives a module of fusion Φ_1 between the automorphic forms on $\mathbf{B}^\times$ having a K_p-fixed vector ($K_p \simeq GL(2, \mathbf{Z}_p)$ a maximal compact subgroup of $B_p^\times$) and those forms which have an I_p-fixed vector but no K_p-fixed vector, with $I_p \subset K_p$ the Iwahori subgroup.

Next we use a refined version of the Eichler/Shimizu/Jacquet-Langlands correspondence to show Φ_1 is also a module of fusion between automorphic forms of $GL(2)$. We conclude that our module is capable of detecting all congruences modulo most primes.

We now describe the contents of this paper in more detail. In Chapter 1 we use the language of Jacquet and Langlands, partly to emphasize the natural context for generalizations of the results. Denote by $\mathbf{A}$ the adèles of $\mathbf{Q}$. Let $B/\mathbf{Q}$ be a quaternion algebra, let $\mathbf{B}^\times$ be the algebraic group attached to the multiplicative group $B^\times$ of B, and let $\mathscr{A}(\mathbf{B}^\times, \omega)$ be the space of automorphic forms on $\mathbf{B}^\times(\mathbf{A})$

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