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UNIQUENESS THEOREMS THROUGH THE METHOD OF MOVING SPHERES

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1. Introduction. For $n \geqslant 2$, $R > 0$, $\bar{x} \in \mathbb{R}^n$, let

$$\mathbb{R}^n_+ = \{(x_1, \dots, x_{n-1}, t) | (x_1, \dots, x_{n-1}) \in \mathbb{R}^{n-1}, t > 0\},$$

$$B_R(\bar{x}) = \{x \in \mathbb{R}^n | |x - \bar{x}| < R\}, \qquad B_R = B_R(0),$$

$$B_R^+(\bar{x}) = \{(x_1, \dots, x_{n-1}, t) \in B_R(\bar{x}) | t > 0\}, \qquad B_R^+ = B_R^+(0).$$

We always use the notation $x = (x', t) \in \mathbb{R}^n_+$.

For $n \geqslant 3$, $c \in \mathbb{R}$, we consider

$$\begin{cases} -\Delta u = n(n-2)u^{(n+2)/(n-2)} & \text{in } \mathbb{R}^n_+, \\ \dfrac{\partial u}{\partial t} = cu^{n/(n-2)} & \text{on } \partial\mathbb{R}^n_+. \end{cases} \tag{1}$$

It is easy to check that for all $\varepsilon > 0$, $x_0' \in \mathbb{R}^{n-1}$, and $t_0 = (n-2)^{-1}\varepsilon c$, the following functions are solutions of (1):

$$u(x', t) = \left(\frac{\varepsilon}{\varepsilon^2 + |(x', t) - (x_0', t_0)|^2}\right)^{(n-2)/2}. \tag{2}$$

THEOREM 1.1. *Let $u \in C^2(\mathbb{R}^n_+) \cap C^1(\overline{\mathbb{R}}^n_+)$ $(n \geqslant 3)$ be any nonnegative solution of (1). Then either $u \equiv 0$ or u takes the form (2) for some $\varepsilon > 0$, $x_0' \in \mathbb{R}^{n-1}$, and $t_0 = (n-2)^{-1}\varepsilon c$.*

Almost the same proof applies to the following equation for $n \geqslant 3$.

$$\begin{cases} -\Delta u = 0 & \text{in } \mathbb{R}^n_+, \\ \dfrac{\partial u}{\partial t} = cu^{n/(n-2)} & \text{on } \partial\mathbb{R}^n_+. \end{cases} \tag{3}$$

When $c < 0$, for any $\varepsilon > 0$, $x_0' \in \mathbb{R}^{n-1}$ and $t_0 = -(n-2)^{-1}\varepsilon c$, the following

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