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DRESSING ORBITS OF HARMONIC MAPS

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Introduction. At the heart of the modern theory of harmonic maps from a Riemann surface to a Riemannian symmetric space is the observation that, in this setting, the harmonic map equations have a zero-curvature representation [19], [24], [28] and so correspond to loops of flat connections. This fact was first exploited in the mathematical literature by Uhlenbeck in her study [24] of harmonic maps $\mathbb{R}^2 \to G$ into a compact Lie group G. Uhlenbeck discovered that harmonic maps correspond to certain holomorphic maps, the *extended solutions*, into the based loop group ΩG, and used this to define an action of a certain loop group on the space of harmonic maps. However, the main focus of [24] was on harmonic maps of a two-sphere, and for these maps the action reduces to an action of a finite-dimensional quotient group (see also [1], [9]).

In another direction, the zero-curvature representation has been central to recent progress in the understanding of the harmonic map equations as soliton equations, i.e., as completely integrable Hamiltonian ordinary differential equations. By solving certain Lax flows on loop algebras, a rather complete description of all harmonic tori in symmetric spaces and Lie groups has been obtained. Of particular importance in this approach are the harmonic maps of finite type: these arise from Lax flows on finite-dimensional subspaces of loop algebras and correspond to linear flows on Jacobians of certain algebraic curves [2], [3], [4], [8], [10], [18]. Among these harmonic maps, we further distinguish those of *semisimple* finite type (see Section 1.3 below), which are characterised by a semisimplicity condition on their derivative. Semisimple finite-type harmonic maps account for all nonconformal harmonic tori in rank-one symmetric spaces of compact type [4], all nonisotropic harmonic tori in spheres and complex projective spaces [3], and all doubly periodic solutions to the abelian affine Toda field equations for simple Lie groups [2].

The purpose of this paper is to describe some interactions between these two approaches. Our starting point is the fact that underlying all of the above results is the existence of Iwasawa-type decompositions of the loop groups and algebras concerned. On the one hand, the Lax equations mentioned above arise from an Iwasawa decomposition of certain twisted loop algebras via the Adler-Kostant-Symes scheme [5]. On the other hand, the loop group action of Uhlenbeck is

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