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TRANSVERSALITY IN ELLIPTIC MORSE THEORY FOR THE SYMPLECTIC ACTION

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CONTENTS

1. Introduction 251
2. The Carleman similarity principle 253
3. Unique continuation 257
4. Injective points 258
5. Transversality 263
6. Symmetric matrices 273
7. Transversality for simple curves 278
8. Equivariant action functional 288

Introduction. Our goal in this paper is to settle some transversality questions for the perturbed nonlinear Cauchy-Riemann equations on the cylinder. These results play a central role in the definition of symplectic Floer homology and hence in the proof of the Arnold conjecture. There is currently no other reference to these transversality results in the open literature. Our approach does not require Aronszajn's theorem. Instead we derive the unique continuation theorem from a generalization of the Carleman similarity principle.

Let (M, ω) be a compact symplectic manifold and consider the differential equation

$$\dot{x}(t) = X_t(x(t)), \tag{1}$$

where $X_t = X_{t+1}: M \to TM$ is a smooth family of symplectic vector fields, i.e., the 1-forms $\iota(X_t)\omega$ are closed. The periodic solutions $x(t) = x(t+1)$ of (1) are the zeros of the closed 1-form Ψ_X on the loop space $\mathscr{L}$ of M defined by

$$\Psi_X(x; \xi) = \int_0^1 \omega(\dot{x}(t) - X_t(x(t)), \xi(t))\, dt$$

for $\xi \in T_x\mathscr{L} = C^\infty(x^*TM)$. On the universal cover of $\mathscr{L}$, this 1-form is the differential $\Psi_X = d\mathscr{A}_X$ of the symplectic action functional $\mathscr{A}_X: \tilde{\mathscr{L}} \to \mathbb{R}$. We shall assume throughout that the periodic solutions of (1) are all nondegenerate. This is equivalent to the condition that $\mathscr{A}_X$ is a Morse function.

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