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POLARISATIONS OF TYPE (1, 2, ..., 2) ON ABELIAN VARIETIES

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To M. S. Narasimhan and C. S. Seshadri on their sixtieth birthdays.

§1. Introduction. Not much seems to be known about the geometry of abelian varieties with nonprincipal polarisations. The algebraic equivalence class of any ample line bundle on an abelian variety A of dimension r determines an r-tuple $\delta = (\delta_1, \delta_2, \ldots, \delta_r)$ of positive integers with $\delta_1 | \delta_2 | \cdots | \delta_r$. This is called the *type* of the line bundle. One might wish to understand at least the generic behaviour of the linear system of τ, where τ is a line bundle of type δ on an abelian variety.

The case $r = 2$ has been discussed in recent years by many authors. If $\delta_1 \geqslant 3$, then the linear system is *always* very ample by the classical result of Lefschetz. If $\delta_1 = 2$, then it is generically very ample if $\delta_r > 2$ [20, Theorem 4.5]. If $\delta = (2, 2, \ldots, 2)$, then generically the linear system imbeds the *Kummer variety* $\mathscr{K} = A/\imath$, where $\imath$ is the natural involution $x \mapsto -x$ of A.

We are primarily interested here in the case $\delta = (1, 2, \ldots, 2)$. First, in this case, there is a base locus consisting of $2^{2(r-1)}$ points. The linear system gives a morphism of the variety σA obtained by blowing up these points. Moreover, it is invariant under the natural involution $\imath$ (upon a proper choice of the line bundle in the algebraic equivalence class). If $r \geqslant 4$, we show that the linear system imbeds $\sigma A/\imath$ at least birationally.

We prove this by considering abelian varieties arising from the following construction. Let C be a (smooth projective) curve and $\tilde{C}$ a 2-sheeted covering of C ramified at four points. Then the Prym variety P is an abelian variety with a polarisation of type $(1, 2, \ldots, 2)$. We study the rational map given rise to by this linear system by bringing into play the moduli of vector bundles on C of rank 2 and trivial determinant. In this case, (assuming that C is of genus at least 4), we show that the variety $\sigma(P)/\imath$ is imbedded birationally but that there is actually a surface over which the mapping is $(2:1)$. Moreover, the data consisting of $\tilde{C}$, C, etc. can be recovered from the geometry of this imbedding, thus providing a Torelli-type theorem for these Pryms. (Compare Donagi-Smith [8].)

By an obvious degeneracy argument, this proves the result for generic abelian varieties claimed above. However, it is possible that the linear system behaves even better in the case of generic abelian varieties since the varieties that we have considered above are quite special.

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