Vol. 80, No. 1 DUKE MATHEMATICAL JOURNAL October 1995

A DESCENT PROBLEM FOR QUADRATIC FORMS

BRUNO KAHN

Let F be a field of characteristic $\neq 2$. Some important problems in the algebraic theory of quadratic forms are to determine when an anisotropic quadratic form over F becomes isotropic or hyperbolic over the function field of a quadric [8], [10], [11]. We consider here a higher, related analogue of these problems: when is a quadratic form over the function field of an F-quadric defined over F?

More specifically, let q be an anisotropic quadratic form over F, $K = F(q)$ the function field of the projective hypersurface X of equation $q(x) = 0$ and φ a quadratic form over K. Recall the *unramified Witt ring* $W_{nr}(K/F) = W_{nr}(K)$, consisting of those Witt classes in $W(K)$ whose second residues at each codimension 1 point of X are 0 [23]. The purpose of this paper is to discuss the following conjecture.

CONJECTURE 1. *Assume that*

- $\dim \varphi < (1/2)\dim q$;
- $\varphi \in W_{nr}(K)$.

Then φ is defined over F.

In general, we cannot prove this conjecture. However, we shall prove here the following theorem.

THEOREM 1. *Conjecture 1 holds if the following hold.*

- $\dim \varphi \leqslant 5$.
- $\dim \varphi = 6$ *and φ is an Albert form. In this case, φ is defined over F by an Albert form.*
- $\dim \varphi = 8$ *and φ is similar to a Pfister form. In this case, φ is defined over F by a form similar to a Pfister form.*

(An *Albert form* is a 6-dimensional form with trivial discriminant.)

In fact, we have a stronger result than Theorem 1, and also partial results for some other φ's.

Theorem 2. *With notation as above, for conjecture 1 to hold, it is enough that*

(a) $\dim \varphi = 1$:	$\dim q > 2$.
(b) $\dim \varphi = 2$:	$\dim q > 4$ *or* $\dim q = 4$, $d_{\pm} q \notin \{1, d_{\pm}\varphi\}$.
(c) $\dim \varphi = 3$:	$\dim q > 6$ *or* $\dim q = 6$, $d_{\pm} q \neq 1$.

Received 18 November 1994. Revision received 24 March 1995.