Vol. 80, No. 1 DUKE MATHEMATICAL JOURNAL © October 1995

FAKE 3-CONNECTED COVERINGS OF LIE GROUPS

J. AGUADÉ, C. BROTO, AND M. SANTOS

1. Introduction. In [1] it was proved that for each prime p there are infinitely many fake 3-connected coverings of S^3. By "fake" we mean spaces with the same mod p cohomology as $S^3\langle 3\rangle$ (as algebras over the Steenrod algebra) but different p-completed homotopy type. After that work was completed one could wonder if the existence of such fake spaces was a general phenomenon, and, in particular, if one could use the same methods to produce fake three connected coverings of other Lie groups beside S^3. In this paper we prove that the results of [1] cannot be extrapolated since, indeed, there is homotopy uniqueness up to p-completion for 3-connected coverings of several compact connected Lie groups and p-compact groups.

If p is a regular prime for the compact connected Lie group G, then S^3 is a direct factor of G at the prime p, and one can trivially construct infinitely many fake $G\langle 3\rangle$ out of the fake $S^3\langle 3\rangle$ constructed in [1]. If p is quasi-regular for G in the sense of [10], then G splits at p as a product of odd-dimensional spheres and spaces $B_n(p)$ that are sphere bundles over spheres. Hence, in this more general situation, fake $G\langle 3\rangle$ would arise if there are fake $B_3(p)\langle 3\rangle$. The main result of this paper shows that there are no such fakes.

THEOREM 1. *Let $B(p)$ denote the S^3-bundle over S^{2p+1} classified by a generator of the p-component of $\pi_{2p}S^3$. Let X be such that $H^*(X;\mathbb{F}_p)\cong H^*(B(p)\langle 3\rangle;\mathbb{F}_p)$ as algebras over the Steenrod algebra. Then $\hat{X}_p\simeq B(p)\langle 3\rangle^{\wedge}_p$.*

COROLLARY 2. *Up to p-completion, there is only one space with the same mod p cohomology (as algebras over the Steenrod algebra) as $G\langle 3\rangle$ if G is (i) $SU(3)$ for $p=2$, (ii) $Sp(2)$ for $p=3$, or (iii) G_2 for $p=5$.*

The proof of the main result is obtained by an analysis similar to the one done in [1] for the case of S^3. Starting with a space X with the same mod p cohomology as $B(p)\langle 3\rangle$, we construct an infinite tower

$$X\to X_1\to X_2\to X_3\to\cdots$$

such that X_{i+1} is obtained from X_i essentially by dividing by an action of the group $B\mathbb{Z}/p$. At the far right end of the tower we get a space which up to p-completion coincides with $B(p)$, and so we get a map $X\to B(p)$ which induces

Received 4 November 1994. Revision received 21 February 1995.
The authors are partially supported by DGICYT grant PB91-0467.