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THE MULTIPLICATIVE ANOMALY FOR DETERMINANTS OF ELLIPTIC OPERATORS

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Introduction. Let M be a compact n-dimensional manifold without boundary and let E be a complex vector bundle over M of rank N. Write $C^\infty(E)$ for the smooth sections of E. Suppose $B: C^\infty(E) \to C^\infty(E)$ is a pseudodifferential operator which is polyhomogeneous of degree β, i.e., (i) and (ii) below are satisfied.

(i) In any local coordinates over which E is trivial,

$$\sigma(B)(x, \xi) \sim \sum_{j=0}^{\infty} b_j(x, \xi), \tag{0.1}$$

where $b_j(x, \xi)$ is an $N \times N$-matrix-valued function which is homogeneous in ξ of degree $\beta - j$.

(ii) There exist local coordinates in which $b_0(x, \xi)$ is not identically zero.

Write $\sigma_0(B)$ for the principal symbol of B, which is a section of the bundle $\mathrm{End}(E)$ over the manifold $T^*(M)\backslash 0$. We say that B has *principal angle* θ, if $\sigma_0(B)(x, \xi)$ has no eigenvalue on the ray $R_\theta = \{re^{i\theta}: r \geqslant 0\}$ for any $(x, \xi) \in T^*(M)\backslash 0$; this is clearly stronger than being elliptic.

Let B be a polyhomogeneous pseudodifferential operator of degree $\beta > 0$ on E and suppose that B has principal angle θ. Then B has discrete spectrum, and each point λ in the spectrum is an eigenvalue whose space of generalized eigenfunctions (functions annihilated by some power of $(\lambda I - B)$) is finite-dimensional. Let $\lambda_1, \lambda_2, \lambda_3, \ldots$ be the list of the nonzero spectrum of B repeated according to multiplicity. Then

$$Z(s) = \sum_{j=1}^{\infty} \lambda_j^s \tag{0.2}$$

converges for $\Re s < -n/\beta$. Here,

$$\lambda^s = |\lambda|^s e^{is \arg \lambda}, \qquad \theta \leqslant \arg \lambda < \theta + 2\pi. \tag{0.3}$$

By Lidskii's theorem and [18], $Z(s)$ can be meromorphically extended to all complex s, and is regular at $s = 0$. The *zeta-regularized determinant* $\det' B$ is defined by

$$\log \det' B = Z'(0).$$

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