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ISOGENIES OF FORMAL GROUP LAWS AND POWER OPERATIONS IN THE COHOMOLOGY THEORIES E_n

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1. Introduction

1.1. Overview. This paper has three parts. The first part is about finite isogenies or subgroups of formal groups as studied by Lubin [Lub67]. Its main result is the construction of coordinates on certain Lubin-Tate formal groups which are preserved under isogenies. It is written largely without reference to algebraic topology, to highlight the formal group-theoretic result (Theorem 4), which we hope is of independent interest.

The problem addressed by Theorem 4 arose in our study of cohomology operations in elliptic cohomology and the cohomology theories E_n, which comprises the second part of the paper. Recent work [DHS88, Hop87, Wit88, BT89] has shown these theories to be of considerable interest. They are all *Landweber exact*, which means that they can be constructed from the complex cobordism functor MU by means of a genus

$$MU^* \xrightarrow{t} E^*$$

such that the resulting functor

$$E^*(X) \overset{\text{def}}{=} E^* \underset{MU^*}{\otimes} MU^*(X) \tag{1.1.1}$$

is a cohomology theory on the category of finite complexes.

It would be very pleasant to have good geometric or analytic descriptions of these theories, along the lines of ordinary rational homology or K-theory (which are the initial cases E_0 and E_1). One expects that a good geometric description of a cohomology theory will provide a wealth of additional informaton. For example, one expects symmetries of the geometry to give rise to cohomology operations. We have turned this idea upside down and examined cohomology operations—in particular, power operations—in hopes of learning about the conjectural geometry of the theories E_n.

Our original aim was to imitate Atiyah's construction of "power operations" in K-theory [Ati66], using the work of Hopkins, Kuhn, and Ravenel [HKR92] in place of the representation theory of the symmetric group. The reason for singling

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