Vol. 79, No. 1 DUKE MATHEMATICAL JOURNAL © July 1995

ON ENERGY MINIMIZING MAPPINGS BETWEEN AND INTO SINGULAR SPACES

JINGYI CHEN

1. Introduction. In a remarkable work [GS], M. Gromov and R. Schoen utilize the variational method to establish a theory of harmonic mappings into nonpositively curved Riemannian simplicial complexes from smooth manifolds. Furthermore, they apply this theory to study the p-adic superrigidity for lattices in groups of rank one. The analytic aspect of their work generalizes the earlier results on existence and uniqueness in a fixed homotopy class of harmonic maps in the sense that the target may have singular points.

In Section 2 of this paper, we consider the energy minimizing mappings from singular spaces into nonpositively curved singular spaces. We show that the energy minimizing mappings are Hölder continuous provided the domain space is a locally $(n-1)$-connected, locally finite n-dimensional Riemannian simplicial complex. (See the definitions in Section 2.) We also illustrate by examples that this isoperimetric condition is necessary in order to have the Hölder continuity. The main theorem in Section 2 is the following.

THEOREM 1.1. *Suppose that a locally finite n-dimensional Riemannian simplicial complex X is locally $(n-1)$-connected, Y is a locally finite Riemannian simplicial complex with nonpositive curvature, and $u: X \to Y$ is energy minimizing with image lying in a compact subset of Y. Then u is Hölder continuous in the interior of X.*

In Section 3, we study the case when the domain space is a smooth Riemannian manifold with nonnegative sectional curvatures, and the image space is a locally finite Riemannian simplicial complex which is nonpositively curved. We state our main results below.

THEOREM 1.2. *Suppose that M is a complete Riemannian manifold with nonnegative sectional curvatures and Y is a nonpositively curved locally finite Riemannian simplicial complex. Let $u: M \to Y$ be an energy minimizing mapping with image lying in a compact subset of Y. Then the energy density $v = |\nabla u|^2$ is subharmonic in the weak sense.*

THEOREM 1.3. *Suppose that M is a complete Riemannian manifold with nonnegative sectional curvatures and Y is a nonpositively curved locally finite Riemannian simplicial complex. If $u: M \to Y$ is an energy minimizing mapping with image lying in a compact subset of Y, then there is a constant C depending only on $n = \dim M$*

Received 22 September 1993. Revision received 11 October 1994.