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SMOOTH GROUP ACTIONS ON DEFINITE 4-MANIFOLDS AND MODULI SPACES

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In this paper we give an application of equivariant moduli spaces to the study of smooth group actions on certain 4-manifolds. A rich source of examples for such actions is the collection of algebraic surfaces (compact and nonsingular) together with their groups of algebraic automorphisms. From this collection, further examples of smooth but generally nonalgebraic actions can be constructed by an equivariant connected sum along an orbit of isolated points. Given a smooth oriented 4-manifold X which is diffeomorphic to a connected sum of algebraic surfaces, we can ask: (i) which (finite) groups can act smoothly on X preserving the orientation, and (ii) how closely does a smooth action on X resemble some equivariant connected sum of algebraic actions on the algebraic surface factors of X?

For the purposes of this paper we will restrict our attention to the simplest case, namely $X = P^2(\mathbf{C}) \# \cdots \# P^2(\mathbf{C})$, a connected sum of n copies of the complex projective plane (arranged so that X is simply connected). Furthermore,

ASSUMPTION. *All actions will be assumed to induce the identity on $H_*(X, \mathbf{Z})$.*

In previous works [17], [18], [19], we considered problem (i) and a variant of problem (ii) when $X = P^2(\mathbf{C})$. It turned out that the only finite groups which could act as above on $P^2(\mathbf{C})$ were the subgroups of $PGL_3(\mathbf{C})$ ([18] and [23] independently). For problem (ii) there are 2 interesting notions weaker than smooth equivalence. If (X, π) is a smooth action, then the isotropy group $\pi_x = \{g \in \pi | gx = x\}$, $x \in X$, acts linearly on the tangent space $T_x X$ and we can ask the following.

Question (iii) a. Given an action (X, π), is there an equivariant connected sum of actions on $P^2(\mathbf{C})$ with the same fixed point data and tangential isotropy representations?

Question (iii) b. Given an action (X, π), is there an equivariant connected sum of actions on $P^2(\mathbf{C})$ which is π-homotopy equivalent or π-equivariantly homeomorphic to (X, π)?

Partial results were obtained on these questions in [17] and [10]: if π acts smoothly on $P^2(\mathbf{C})$, inducing the identity on homology, and the action has an

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