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DISTRIBUTION OF RESONANCES FOR THE NEUMANN PROBLEM IN LINEAR ELASTICITY OUTSIDE A STRICTLY CONVEX BODY

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1. Introduction. Let $\mathcal{O}$ be a strictly convex compact set in $\mathbf{R}^3$ with C^∞-smooth boundary Γ and denote by $\Omega = \mathbf{R}^3 \setminus \mathcal{O}$ the exterior domain. Denote by Δ_e the elasticity operator, which is a 3×3 matrix-valued differential operator defined by

$$\Delta_e v = \mu_0 \Delta v + (\lambda_0 + \mu_0)\nabla(\nabla \cdot v),$$

$v = {}^t(v_1, v_2, v_3)$. Here λ_0, μ_0 are the Lamé constants and we assume that

$$\mu_0 > 0, \quad 3\lambda_0 + 2\mu_0 > 0. \tag{1.1}$$

The Neumann boundary conditions for Δ_e are of the form

$$\sum_{j=1}^{3} \sigma_{ij}(v)\nu_j|_\Gamma = 0, \quad i = 1, 2, 3, \tag{1.2}$$

where $\sigma_{ij}(v) = \lambda_0 \nabla \cdot v \delta_{ij} + \mu_0(\partial v_i/\partial x_j + \partial v_j/\partial x_i)$ is the stress tensor, and ν is the outer normal to $\Gamma = \partial\Omega$. It is known that $-\Delta_e$, acting on functions $v \in C^\infty_{\text{comp}}(\overline{\Omega}; \mathbf{C}^3)$ satisfying (1.2), can be extended to a selfadjoint operator on $L^2(\Omega; \mathbf{C}^3)$ which will be denoted by L. The operator L is nonnegative and has no point spectrum. Then the cut-off resolvent $R_\chi(\lambda) = \chi(L - \lambda^2)^{-1}\chi$, $\chi \in C_0^\infty$ being a cut-off function equal to 1 near Γ, can be extended as a meromorphic function from $\operatorname{Im} \lambda < 0$ to the whole complex plane $\mathbf{C}$ with possible poles in $\operatorname{Im} \lambda > 0$ (see, e.g., [Va], [Vo]). The poles of $R_\chi(\lambda)$ are called *resonances* (known also as scattering poles).

There are a lot of works dealing with resonances for the Dirichlet or Neumann Laplacian in an exterior domain. It follows from [MS1] and [MS2] that if there are no trapped rays, the singularities of the solution of the wave equation escape to infinity. Thus the method in [LP2] (see also [Va]) gives that, for nontrapping obstacles (and in particular for strictly convex ones), for any $C_1 > 0$ there exists $C_2 > 0$ (depending on C_1) so that all the resonances are above the curve $\operatorname{Im} \lambda = C_1 \ln|\lambda| - C_2$. In the case of analytic boundary this was improved in [BLR] to a cubic curve $\operatorname{Im} \lambda = C_1 |\lambda|^{1/3} - C_2$ with some constants $C_1, C_2 > 0$ which can be calculated explicitly. Recently, it was shown in [SZ] and [HL] that this is the

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