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v_n TELESCOPES AND THE ADAMS SPECTRAL SEQUENCE

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1. Introduction. Recall from [9] that any finite spectrum X admits a v_n map $v: \sum^k X \to X$. We remind the reader that to say v is a v_n map means that $K(i)_*(v) = 0$ if $i \neq n$ and $K(n)_*(v)$ is an isomorphism. ($K(i)$ is the ith Morava K-theory.) The spectrum X determines for what value of n X has a v_n map; the value of n is the smallest integer for which $K(n)_*(X) \neq 0$. Such an X will be said to be of *type n*.

Given X of type n and $v: \sum^k X \to X$ a v_n map, we define $f \in \pi_* X$ to be v_n *torsion* if $v^k \circ f \simeq *$ for some k, i.e., if f is in the kernel of the map

$$\pi_* X \to \varinjlim_v \pi_* X = v^{-1}\pi_* X = \pi_*\left(\varinjlim_v X\right).$$

By the essential uniqueness of v_n maps in [6], this definition is independent of any choice involved in the map v.

One statement of the telescope conjecture is that when X is as above, $L_n X = \varinjlim_v X$. Here L_n is the usual Bousfield localization [4, Theorem 1.1] with respect to the Johnson-Wilson theory $E(n)$, or equivalently with respect to the wedge of Morava K-theories $K(0) \vee K(1) \vee \cdots \vee K(n)$. So in the cases where the telescope conjecture holds, our definition of v_n torsion in the homotopy groups of X is the same as the kernel of the map

$$\pi_* X \to \pi_* L_n X.$$

This note grew out of an attempt to understand the relationship between two notions of "v_n-torsion" (on spectra that do not necessarily admit v_n maps), one that is rather geometric and related to the definition given above (in the case where the spectrum in question is of type n), and another involving the functor L_n that seems more canonical. The more geometric definition follows.

Definition 1.1. A map $f \in \pi_*(X)$ will be said to be v_n-torsion if f can be factored

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