Vol. 77, No. 3 DUKE MATHEMATICAL JOURNAL © March 1995

LOCAL INDICES OF p-ADIC DIFFERENTIAL OPERATORS CORRESPONDING TO ARTIN-SCHREIER-WITT COVERINGS

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Introduction. In this paper we study local indices of p-adic differential operators derived from p-adic representations. It has been well known that there exists a strong analogy between the theory of $\mathscr{D}$-modules and the theory of Galois representations. In this analogy, regular singularities correspond to tame ramifications and irregularities correspond to wild ramifications. On the other hand, for a discrete valuation field or a variety in characteristic $p > 0$, we can construct modules with connection, and as a result we get p-adic $\mathscr{D}$-modules and p-adic differential operators, from p-adic representations of the Galois group or the fundamental group [Be], [Cr], [Fo]. It is a kind of analogy of Riemann-Hilbert correspondence on a complex manifold, though in our case a p-adic representation may give rise to an irregular singularity. It is natural to conjecture that if a p-adic representation has finite local monodromy, its Swan conductor is equal to the irregularity of the corresponding module with connection. In [Ro], Robba defined the local index for a p-adic differential operator and used it to define the irregularity. The main result of this paper is that, for a complete discrete valuation ring of characteristic $p > 2$ with perfect residue field, the Swan conductor of a representation of rank 1 is equal to the local index of the corresponding p-adic differential operator (5.5), as conjectured by Crew [Cr, 4.16.1]. We will not consider $\mathscr{D}$-modules here, but I hope that the calculation of this paper would be useful to compute irregularities of $\mathscr{D}$-modules.

The author would like to express his sincere gratitude to Professor Kazuya Kato and Professor Takeshi Saito for their advice and encouragement. He would also like to thank Kazuhiro Fujiwara, Yuuichirou Taguchi, and Nobuo Tsuzuki for useful discussions.

Notation. Throughout this paper we fix an odd prime p. We denote by $\mathbf{Z}_p$ the p-adic completion of $\mathbf{Z}$ endowed with the nonarchimedean valuation normalized by $|p| = p^{-1}$. For a commutative ring A, we denote by $W_n(A)$ the ring of Witt vectors of length n in A and define $W(A)$ as the inverse limit of $W_n(A)$.

1. ϕ_m, ψ_m, θ_m. In this section we define several functions which will be used for the Kummer-Artin-Schreier-Witt theory in Section 3.

(1.1) For a commutative ring R over $\mathbf{Z}_p$, we consider the following conditions:

(1.1.1) R is equipped with a nonarchimedean valuation extending that of $\mathbf{Z}_p$.

Received 8 March 1994. Revision received 12 September 1994.