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MAXIMAL OPERATORS ASSOCIATED TO FAMILIES OF FLAT CURVES IN THE PLANE

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Introduction. Let C denote a smooth curve in the plane. Let $M_t f(x) = \int_C f(x - ty)\, d\sigma(y)$, where $d\sigma(y)$ denotes a cutoff function times the Lebesgue measure on C. Let $\mathcal{M}f(x) = \sup_{t>0} M_t f(x)$. A question we ask is for what range of the exponents p is the following a priori inequality satisfied:

$$\|\mathcal{M}f\|_p \leqslant C_p \|f\|_p, \qquad f \in \mathcal{S}. \tag{1.1}$$

Bourgain showed that if C has nonvanishing curvature, the inequality (1.1) holds for $p > 2$ (see [B]). In this paper, we shall consider a situation when the curvature is allowed to vanish of finite order on a finite set of isolated points. We shall need the following definition.

Definition 1.1. Let $C: I \to \mathbb{R}^2$, where I is a compact interval in $\mathbb{R}$ and C is smooth. We say that C is of finite type if $\langle (C(x) - C(x_0)), \mu \rangle$ does not vanish of infinite order for any $x_0 \in I$, and any unit vector μ.

We shall also need a more precise definition which would specify the order of vanishing at each point. Let a_0 denote a point in the compact interval I. We can always find a smooth function γ, such that in a small neighborhood of a_0, $C(s) = (s, \gamma(s))$, where $s \in I$.

Definition 1.2. Let C be defined as before. Let $C(s) = (s, \gamma(s))$ in a small neighborhood of a_0. We say that C is of finite type m at a_0 if $\gamma^{(k)}(a_0) = 0$ for $1 \leqslant k < m$, and $\gamma^{(m)}(a_0) \neq 0$.

Our main result is the following.

THEOREM 1.1. *Let C be a finite-type curve which is of finite type m at a_0. Let $M_t f(x) = \int_C f(x - ty)\, d\sigma(y)$, where $d\sigma$ is the Lebesgue measure on C multiplied by a smooth cutoff function supported in a sufficiently small neighborhood of a_0. Let $\mathcal{M}f(x) = \sup_{t>0} M_t f(x)$. Then*

$$\|\mathcal{M}f\|_p \leqslant C_p \|f\|_p \qquad \text{for } p > m. \tag{1.2}$$

Furthermore, the result is sharp. Let $h_p(x) = |x_2|^{1/p} \log(1/|x_2|)^{-1} \phi(x)$, where $\phi(x)$ is a nonnegative C_0^∞ function supported in the unit ball, such that $\phi \equiv 1$ near the

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