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# THE TWISTED ENDOSCOPY OF $GL(4)$ AND $GL(5)$: TRANSFER OF SHALIKA GERMS

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**Introduction.** Kottwitz and Shelstad have general conjectures relating twisted orbital integrals on a reductive $p$-adic group to stable orbital integrals on endoscopic groups [KS]. The theory of descent, currently known only in the special case of standard endoscopy, reduces the transfer of orbital integrals to a local statement about the matching of Shalika germs at the identity, for the original reductive group together with the additional reductive groups obtained by descent [LS2].

This paper proves the needed statements about the matching of Shalika germs at the identity for the groups $GL(4)$ and $GL(5)$. Thus, the results of this paper, coupled with an expected theory of descent, would imply the transfer of twisted orbital integrals on $GL(4)$ and $GL(5)$ to stable orbital integrals on endoscopic groups.

These calculations give a first example of transfer in a *nonelementary* setting. The Shalika germs of the twisted orbital integrals on $GL(4)$ and $GL(5)$ are expressed by the number of points on a family of elliptic curves over finite fields. The stable orbital integrals on the endoscopic groups $SO(5)$ and $Sp(4)$ have a similar description. The transfer of orbital integrals to the endoscopic groups is established by producing isogenies between the families of elliptic curves. The presence of elliptic curves in a similar context was first noticed by Kazhdan, Lusztig, and Bernstein [KLB]. For further results along these lines, see [H3].

These calculations give complete formulas for the Shalika germs of $Sp(4)$. By results of Kazhdan and Harish-Chandra, the Fourier transforms of subregular nilpotent orbits on the elliptic set coincide, up to a change of basis, with cuspidal combinations of subregular Shalika germs on the group $Sp(4)$ (see [Ka], [HC]). Thus the Fourier transforms of certain subregular nilpotent orbits on elliptic elements of $Sp(4)$, being expressed in terms of points on families of elliptic curves over finite fields, are not elementary.

The transfer of orbital integrals and the related fundamental lemmas are essential steps in the development of a stable trace formula. This paper brings us one step closer to obtaining a stable trace formula for $Sp(4)$.

We also give the first verification of an unpublished conjecture of Assem and Kottwitz. Their conjecture states that the stable Shalika germ associated with a unipotent conjugacy class that is not special is identically zero. (For the definition of *special* unipotent classes, see [C].) We verify this conjecture for the *two-regular*

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