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THE QUANTUM GROUP OF PLANE MOTIONS AND THE HAHN-EXTON q-BESSEL FUNCTION

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1. Introduction. It is generally expected that quantum groups, originally introduced by Drinfeld [4], Jimbo [8], and Woronowicz [39], provide a natural setting for special functions of q-hypergeometric type in a way similar to the relation between Lie groups and special functions of hypergeometric type as described in, e.g., [37]. This relation between quantum groups and q-special functions started with the calculation of the matrix elements of irreducible unitary representations of the quantum $SU(2)$ group in terms of little q-Jacobi polynomials by Vaksman and Soibelman [34], Masuda et al. [26], and Koornwinder [17]. Since then, several other q-special functions have found an interpretation on a quantum group, sometimes yielding new results or new proofs of old results for these functions. A summary of such results can be found in the review articles by Koornwinder [18], and by Noumi [28] in the case of the quantum $SU(2)$ group.

The representation theory of the group $M(2)$ of distance and orientation preserving motions of the Euclidean plane is closely connected with the theory of Bessel functions, cf. [37, Chapter 4]. In this paper we consider the connections between a basic analogue of the Bessel function, the so-called Hahn-Exton q-Bessel function, and the quantum group of plane motions. Explicitly, the matrix elements of irreducible unitary representations of the quantised universal enveloping algebra are Hahn-Exton q-Bessel functions where the argument consists of elements of the dual Hopf $*$-algebra; cf. §5. Using this fact we will prove Hansen-Lommel orthogonality relations and a Hankel transform for the Hahn-Exton q-Bessel function in §5. Vaksman and Korogodskiĭ [33] have announced some of the results which are also proved in this paper; we will compare our results with the results they obtained.

In a sequel [15] to this paper we describe how other basic analogues of the Bessel functions have an interpretation on the quantum group of plane motions. In particular, we give an interpretation of the Jackson q-Bessel function (cf. [7]) and of basic analogues of the Bessel function which are also basic analogues of the Jacobi functions (cf. [22]) arising in the study of the quantum $SU(1, 1)$ group (cf. [24], [25], [32]). Several properties of these basic Bessel functions have an interpretation on the quantum group of plane motions. In [15] we will give an

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