

DERIVED HEIGHTS AND GENERALIZED MAZUR-TATE REGULATORS

MASSIMO BERTOLINI AND HENRI DARMON

1. Introduction	75
2. Algebraic preliminaries	76
2.1. Notations and conventions	
2.2. Duality	
2.2.1. Local duality	
2.2.2. Global duality	
2.3. Assumptions	
2.4. Preliminary calculations	
3. The generalized regulator	84
3.1. The modules A_S and B_S	
3.2. The regulator in R/R^*	
3.3. The regulator in R/R_1^*	
3.4. Remarks on the generalized regulator	
3.4.1. Relationship with the Mazur-Tate regulator	
3.4.2. Behaviour under norms	
3.5. Generalized regulators for cyclic groups: derived heights	
4. The Mazur-Tate conjectures	98
4.1. The general case	
4.2. Modular symbols	
4.3. Conjectures over quadratic fields	
4.3.1. Real quadratic fields and Heegner cycles	
4.3.2. Imaginary quadratic fields and Heegner points	

1. Introduction. Let E be an elliptic curve defined over a number field K , and let L/K be an abelian extension with Galois group G . In [MT1] and [MT2], B. Mazur and J. Tate have defined a height pairing

$$\langle \cdot, \cdot \rangle_{MT}: E_L(K) \times E(K) \rightarrow G,$$

where $E_L(K)$ is a subgroup of finite index of $E(K)$, consisting of the points of $E(K)$ that are local norms from $E(L)$. Let I denote the augmentation ideal in the integral group ring $\mathbb{Z}[G]$. There is a canonical identification $G = I/I^2$, allowing us to

Received 8 October 1993. Revision received 28 February 1994.
 The first author was partially supported by M.U.R.S.T. and C.N.R. of Italy.
 The second author was partially supported by NSF and NSERC.