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NEGATIVE RICCI CURVATURE AND ISOMETRY GROUP

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1. Introduction. It is now known that negative Ricci curvature does not imply any topological restriction on the underlying manifold (cf. [L]). It does, however, impose geometric restriction by the classical result of Bochner [Bo], namely, the isometry group must be finite. In this paper we consider a quantitative version of Bochner's theorem.

Consider the class of Riemannian n-manifolds satisfying

$$-\Lambda \leqslant \mathrm{Ric} \leqslant -\lambda < 0, \qquad \mathrm{inj} \geqslant i_0, \qquad \mathrm{vol} \leqslant V. \tag{1}$$

(Here the upper bound on the volume is equivalent to an upper bound on the diameter.) By a result of M. Anderson [A] this class is precompact in $C^{1,\alpha}$ topology. We prove the following theorem.

THEOREM 1.1. *Let M_i be a sequence of n-manifolds satisfying (1) and $C^{1,\alpha}$ convergent to a $C^{1,\alpha}$ Riemannian manifold M. Then*

(a) $\#\{\mathrm{Iso}(M)\} < +\infty$.

(b) $\overline{\lim}_{i\to\infty} \#\{\mathrm{Iso}(M_i)\} \leqslant \#\{\mathrm{Iso}(M)\}$.

An immediate consequence of Theorem 1.1 is the following result, which was obtained by Katsuda [K], with an additional assumption that the sectional curvature is bounded from below.

COROLLARY 1.2. *There is a constant $N = N(n, \lambda, \Lambda, i_0, V)$ such that for any n-dimensional Riemannian manifold M satisfying (1), the order of the isometry group* Iso(M) *is smaller than N.*

Remark 1. This result can also be considered as a generalization of the Hurwicz Theorem for hyperbolic surfaces, which gives a (very explicit) bound on the order of the isomerty group in terms of the genus.

Remark 2. If one assumes the sectional curvature bound:

$$-\Lambda \leqslant K < 0,$$

then one can drop the injectivity radius lower bound; see [Ym]. Here one can use

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