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CORRECTION TO "METRIC PINCHING OF LOCALLY SYMMETRIC SPACES"

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The proof of Proposition 11 in [1] is incorrect because we do not know whether γ_5 and γ_6 are geodesic directions. We need to change "finite topological dimension" to "finite Hausdorff dimension" (i.e., assume that X is an Alexandrov space) in the statement of Theorem 4, and proceed as follows. (The statements of the pinching theorems are unchanged.) In the following, "nonsingular point" refers to a point whose tangent space is Euclidean.

LEMMA 1. *Let p be a nonsingular point in an Alexandrov space X. Then there exists an $\varepsilon > 0$ such that for any nonsingular points $x, y \in B(p, \varepsilon)$ and minimal curves γ_1, γ_2 joining x and y, if z_1, z_2 are the midpoints of γ_1, γ_2, respectively, $d(z_1, z_2) < d(x, y)/2$.*

Proof. For simplicity, take the lower curvature bound k for X to be negative. Choose an angle $A > 0$ small enough so that, if Γ_1, Γ_2 are unit geodesics in S_k such that $\alpha(\Gamma_1, \Gamma_2) < A$, then for any $0 < t \leqslant 1$, $d(\Gamma_1(t/2), \Gamma_2(t/2)) < t/2$. The existence of such an A follows easily from the distance-angle monotonicity. We have now reduced the problem to: for any nonsingular points x, y close to p and minimal curves γ_1, γ_2 joining x and y, $\alpha(\gamma_1, \gamma_2) < A$. Choose a finite $A/6$-dense set $\{\beta_1, \ldots, \beta_m\}$ of geodesic directions in $\bar{S}_p = S^{n-1}$ which are the unique minimal curves between their endpoints. Now choose $T > 0$ small enough that for any i, j, $\alpha(\beta_i, \beta_j) - \alpha_k(p; \beta_i(T), \beta_j(T)) < A/6$. Then for any nonsingular point x close enough to p and minimal curves β_i^x from x to $\beta_i(T)$, we also have that $\{\beta_i^x\}$ is $A/6$-dense in $\bar{S}_x = S^{n-1}$ and has the property that $\alpha(\beta_i^x, \beta_j^x) - \alpha_k(x; \beta_i(T), \beta_j(T)) < A/6$. Let γ_1, γ_2 be minimal curves starting at x and choose $\beta_{i_1}^x$ and $\beta_{i_2}^x$ such that $\alpha(\gamma_1, \beta_{i_1}^x) < A/6$ and $\alpha(\gamma_2, \beta_{i_2}^x) < A/6$. Then, for any $t < T$,

$$\begin{aligned}\alpha_k(x; \gamma_1(t), \gamma_2(t)) &\geqslant \alpha_k(x; \beta_{i_1}^x(t), \beta_{i_2}^x(t)) - \alpha_k(x; \beta_{i_1}^x(t), \gamma_1(t)) - \alpha_k(x; \beta_{i_2}^x(t), \gamma_2(t)) \\ &\geqslant \alpha_k(x; \beta_{i_1}^x(T), \beta_{i_2}^x(T)) - A/6 - A/6 > \alpha(\beta_{i_1}^x, \beta_{i_2}^x) - A/2 \\ &\geqslant \alpha(\gamma_1, \gamma_2) - 5A/6.\end{aligned}$$

In other words, if $\alpha(\gamma_1, \gamma_2) > A$, then $\alpha_k(x; \gamma_1(t), \gamma_2(t)) > 0$ and so $\gamma_1(t) \neq \gamma_2(t)$. □

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