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THE TOPOLOGICAL GROUP STRUCTURE OF ALGEBRAIC CYCLES

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1. Introduction. It is a well-known fact that the functor which assigns to a complex algebraic variety X its group $\mathscr{Z}_p(X)$ of algebraic p-cycles is not representable. In some instances one can represent the *effective* p-cycles, as in the case of projective varieties, by their Chow varieties [Sam], although this is a far from complete picture of the group of all cycles.

It is the primary objective of this paper to show that the group $\mathscr{Z}_p(X)$ presents a remarkable behavior from the topological point of view. Here we introduce a natural topology on $\mathscr{Z}_p(X)$, for *any* algebraic variety X over $\mathbb{C}$, and study its functorial properties and the topological structure of $\mathscr{Z}_p(X)$.

A particular instance, and our initial motivation, where a topology on $\mathscr{Z}_p(X)$ is considered is in the study of Lawson homology for quasi-projective varieties, developed in [Law], [Fri], [LF1], and [FrGa]. In the case of quasi-projective varieties, the topology studied in the present work coincides with the one used in previous approaches to Lawson homology. However, our approach here is simpler and more natural.

The topology that we introduce on $\mathscr{Z}_p(X)$ is simply defined as the *finest topology which makes continuous all flat families over smooth base spaces.* This notion naturally leads to the introduction of special classes of continuous maps involving cycle groups, which we call *regular maps*. In the particular case of 0-cycles on a projective variety X, the notion of a regular map from a variety S to $\mathscr{Z}_0(X)$ will coincide (up to passage to rational equivalence) with the one defined by A. Rojtman in [Roj, page 554].

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