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ON THE DRINFELD DOUBLE AND THE HEISENBERG DOUBLE OF A HOPF ALGEBRA

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1. Introduction and notations. Throughout this paper, A will be a unital algebra over a field k, with m, Δ, S, and ε denoting respectively the product, the coproduct, the antipode and the counit of A. The product m is also denoted by $a \otimes b \mapsto ab$. The unit element of A is denoted by 1_A, or simply by 1 when causing no confusion. We use the following Sweedler [Sw] notation to denote the coproducts of an element $a \in A$:

$$\Delta a = a_{(1)} \otimes a_{(2)}$$

$$(\Delta \otimes id)\Delta a = (id \otimes \Delta)\Delta a = a_{(1)} \otimes a_{(2)} \otimes a_{(3)},$$

etc. We will also assume that the antipode map S is invertible. Elements of A will be denoted by $a, b, c, \ldots$ unless otherwise indicated.

Let A^* be a Hopf algebra dual to A. By this, we mean that A^* is a Hopf algebra such that there is a nondegenerate pairing $\langle\ ,\ \rangle$ between A and A^* satisfying

$$\langle ab, x\rangle = \langle a \otimes b, \Delta x\rangle = \langle a, x_{(1)}\rangle\langle b, x_{(2)}\rangle,$$

$$\langle a, xy\rangle = \langle \Delta a, x \otimes y\rangle = \langle a_{(1)}, x\rangle\langle a_{(2)}, y\rangle$$

and

$$\langle 1_A, x\rangle = \varepsilon(x), \qquad \langle a, 1_{A^*}\rangle = \varepsilon(a), \qquad \langle a, S(x)\rangle = \langle S(a), x\rangle$$

where $a, b \in A$, $x, y \in A^*$. Here and elsewhere in this paper, we use, for simplicity, the same letters Δ, S and ε to denote the coproduct, the antipode, and the counit of A^*. Elements of A^* will be denoted by $x, y, z, \ldots$ unless otherwise indicated.

The Hopf algebras A^{op} and A^{coop} are defined as follows:

$$A^{op} = (A, m^{op}, \Delta, S^{-1})$$

$$A^{coop} = (A, m, \Delta^{op}, S^{-1}),$$

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