

INEQUALITY OF BOGOMOLOV-GIESEKER TYPE ON ARITHMETIC SURFACES

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Introduction

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References

Introduction. Let M be an n -dimensional compact Kähler manifold with a Kähler form Φ . For a nonzero torsion free sheaf F on M , we define an average degree $\mu(F, \Phi)$ of F with respect to Φ by

$$\mu(F, \Phi) = \frac{\int_M c_1(F) \wedge \Phi^{n-1}}{\text{rk } F}.$$

Let E be a nonzero torsion-free sheaf on M . E is said to be Φ -stable (resp. Φ -semistable) if, for every subsheaf F of E with $0 \subsetneq F \subsetneq E$, an inequality $\mu(F, \Phi) < \mu(E, \Phi)$ (resp. $\mu(F, \Phi) \leq \mu(E, \Phi)$) is satisfied. If a torsion-free sheaf E of rank r is Φ -semistable, then we have

$$\int_M \left(c_2(E) - \frac{r-1}{2r} c_1(E)^2 \right) \wedge \Phi^{n-2} \geq 0,$$

which is called the Bogomolov-Gieseker inequality (cf. [Bo] and [Gi]). The purpose of this paper is to establish an arithmetic analogue of the above inequality on an arithmetic surface.

MAIN THEOREM. *Let K be an algebraic number field, O_K the ring of integers of K , and $f: X \rightarrow \text{Spec}(O_K)$ an arithmetic surface. Let (E, h) be a Hermitian vector bundle on X . If $E_{\overline{\mathbb{Q}}}$ is semistable on the geometric generic fiber $X_{\overline{\mathbb{Q}}}$ of f , then we have an*

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