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REMOVABLE BOUNDARY SINGULARITIES FOR SOLUTIONS OF SOME NONLINEAR DIFFERENTIAL EQUATIONS

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1. Introduction. Suppose D is a bounded domain in $\mathbf{R}^d$ with a C^2 boundary ∂D and F is a closed subset of ∂D. We investigate a boundary value problem:

$$\begin{cases} \Delta u = u^\alpha & \text{in } D, \\ u = f & \text{on } \partial D \backslash F \end{cases} \tag{1}$$

where $\alpha > 1$ and f is a nonnegative continuous function on ∂D, and we study the condition on F in terms of the Hausdorff dimension under which boundary singularities of a solution of (1) are removable. Throughout this paper a solution of problem (1) always means a nonnegative solution.

Suppose g is an increasing function on an interval $[0, a]$ such that $g(0) = 0$. For any $A \subset \mathbf{R}^d$ and any $0 < \varepsilon \leqslant a$ we set $g\text{-}m_\varepsilon(A) = \inf \sum_i g(r_i)$ where infimum is taken over all countable covering of A by open ball $B_{r_i}(x_i)$ of center x_i and radius $r_i \leqslant \varepsilon$. The Hausdorff measure g-m corresponding to g is defined by the formula $g\text{-}m(A) = \lim_{\varepsilon \to 0} g\text{-}m_\varepsilon(A)$. We denote $\Lambda^s\text{-}m(A)$ the Hausdorff measure of A corresponding to $g(t) = t^s$. The Hausdorff dimension $H\text{-}dim(A)$ is defined as the supremun of s such that $\Lambda^s\text{-}m(A) > 0$.

In Section 2 we interpret (1) as the classical problem:

$$\begin{cases} u \in C^2(D) \text{ and } \Delta u = u^\alpha \text{ in } D, \\ \lim\limits_{x \in D, x \to y} u(x) = f(y) \text{ for all } y \in \partial D \backslash F. \end{cases} \tag{1'}$$

We set $\beta = d - ((\alpha + 1)/(\alpha - 1))$ and $\gamma = (\alpha + 1)/(\alpha - 1)$, and we establish the following.

THEOREM 1. *The boundary value problem (1′) has one parameter family of solutions in the following two cases:*
(A) $d < (\alpha + 1)/(\alpha - 1)$ *and F is not empty;*
(B) $d > (\alpha + 1)/(\alpha - 1)$ *and* $\Lambda^s\text{-}m(F) > 0$ *for some* $\beta < s \leqslant d - 1$.

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