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A GENERALIZATION OF THE RADIATION CONDITION OF SOMMERFELD FOR N-BODY SCHRÖDINGER OPERATORS

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1. Introduction. The radiation condition was introduced by Sommerfeld [24], [25] to derive the uniqueness of the solution of the reduced wave equation in $\mathbf{R}^3$:

$$(-\Delta - \lambda)u = f, \qquad \lambda > 0. \tag{1.1}$$

If f decays sufficiently rapidly, this equation has two solutions $u_\pm$ defined by

$$u_\pm = \frac{1}{4\pi}\int_{\mathbf{R}^3} \frac{e^{\pm i\sqrt{\lambda}|x-y|}}{|x-y|} f(y)\,dy.$$

They behave like

$$u_\pm \sim r^{-1}e^{\pm i\sqrt{\lambda}r}a_\pm(\omega), \qquad \omega = x/r, \qquad r = |x| \to \infty, \tag{1.2}$$

and they satisfy the following conditions at infinity:

$$u_\pm = O(r^{-1}), \qquad \left(\frac{\partial}{\partial r} \mp i\sqrt{\lambda}\right)u_\pm = o(r^{-1}). \tag{1.3}$$

The important fact is that the solution of (1.1) is unique if it satisfies the condition (1.3) for u_+ or u_-. The former is usually called the outgoing radiation condition, since $r^{-1}e^{i\sqrt{\lambda}(r-t)}$ represents an outgoing wave for the wave equation $\partial_t^2 v = \Delta v$, and by a similar reason the latter is called the incoming radiation condition. This condition is closely related to the asymptotic behavior at infinity of the resolvent of $-\Delta$. In fact, we have

$$u_\pm = (-\Delta - \lambda \mp i0)^{-1}f.$$

The theorem of Sommerfeld was made mathematically rigorous by Rellich [20] and Vekua [28], and it was extended to general elliptic operators, first-order systems, and potential scattering by Vainverg [27], Grushin [9], Eidus [4], Shulenberger-Wilcox [23] and Agmon-Hörmander [2]. We begin with reviewing the well-known results for 2-body Schrödinger operators. First we introduce some

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