

THE QUASI-CLASSICAL ASYMPTOTICS OF LOCAL
RIESZ MEANS FOR THE SCHRÖDINGER OPERATOR IN
A STRONG HOMOGENEOUS MAGNETIC FIELD

A. V. SOBOLEV

CONTENTS

1. Introduction. Main result	319
2. Preliminary estimates for the magnetic operator	325
3. Weyl pseudodifferential operators	339
4. Similarity transformations. The Egorov-type theorems	348
5. Quasi-classical asymptotics for functions of a Ψ DO	361
6. Reduction to the normal form. Canonical transformation	370
7. Reduction to the normal form. Fourier integral operators	378
8. Properties of P_0	391
9. Asymptotics of \mathcal{N}_h for the operator P_0	397
10. Proof of the Main Theorem	413
Appendix A	419
Appendix B. Tauberian theorem	426
References	428

1. Introduction. Main result. The Schrödinger operator H_0 in $L^2(\mathbb{R}^3)$, with a homogeneous magnetic field pointed along the x_3 -axis, can be written in the form

$$H_0 = (-ih\partial_{x_1} + \mu x_2)^2 - h^2\partial_{x_3}^2 - h^2\partial_{x_2}^2 - \mu h. \quad (1.1)$$

Here $h > 0$ is the Planck constant and $\mu \geq \mu_0 > 0$ represents the strength of magnetic field. The magnetic vector potential has the form $(-\mu x_2, 0, 0)$. It is well known that the spectrum of H_0 is absolutely continuous, coincides with the half-line $[0, \infty)$, and has an infinite set of thresholds (Landau levels) located at the points $2\mu hk$, $k = 0, 1, 2, \dots$ (see [15]). Note that (1.1) differs from the conventional definition of the Schrödinger operator in magnetic field by the term $-\mu h$. This is done to make the spectrum of H_0 start at $\lambda = 0$.

We are going to study spectral properties of the perturbed Schrödinger operator $H = H_V = H_0 + V$ with a real-valued function V . It is well known (see, for example, [3], [25], [28]), that in the case $V(x) \rightarrow 0$, $|x| \rightarrow \infty$ the essential spectrum of H coincides with that of H_0 (i.e., with $[0, \infty)$) and below $\lambda = 0$ the operator H has in

Received 21 July 1993. Revision received 22 October 1993.

Author is an NSERC fellow.

Author partially supported by NSERC under grants OGP0007901 and OGP0138277.