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# THE QUASI-CLASSICAL ASYMPTOTICS OF LOCAL RIESZ MEANS FOR THE SCHRÖDINGER OPERATOR IN A STRONG HOMOGENEOUS MAGNETIC FIELD

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CONTENTS



**1. Introduction. Main result.** The Schrödinger operator $H_0$ in $L^2(\mathbb{R}^3)$, with a homogeneous magnetic field pointed along the $x_3$-axes, can be written in the form

$$H_0 = (-ih\partial_{x_1} + \mu x_2)^2 - h^2\partial_{x_3}^2 - h^2\partial_{x_2}^2 - \mu h. \tag{1.1}$$

Here $h > 0$ is the Planck constant and $\mu \geqslant \mu_0 > 0$ represents the strength of magnetic field. The magnetic vector potential has the form $(-\mu x_2, 0, 0)$. It is well known that the spectrum of $H_0$ is absolutely continuous, coincides with the half-line $[0, \infty)$, and has an infinite set of thresholds (Landau levels) located at the points $2\mu hk$, $k = 0, 1, 2, \ldots$ (see [15]). Note that (1.1) differs from the conventional definition of the Schrödinger operator in magnetic field by the term $-\mu h$. This is done to make the spectrum of $H_0$ start at $\lambda = 0$.

We are going to study spectral properties of the perturbed Schrödinger operator $H = H_V = H_0 + V$ with a real-valued function $V$. It is well known (see, for example, [3], [25], [28]), that in the case $V(x) \to 0$, $|x| \to \infty$ the essential spectrum of $H$ coincides with that of $H_0$ (i.e., with $[0, \infty)$) and below $\lambda = 0$ the operator $H$ has in

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