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ON THE MAXWELL-KLEIN-GORDON EQUATION WITH FINITE ENERGY

S. KLAINERMAN AND M. MACHEDON

1. The Maxwell-Klein-Gordon equations in $\mathbb{R}^{1+3}$. In this paper we plan to study the evolution of the (3 + 1)-dimensional Maxwell-Klein-Gordon equations in the energy norm. Our main result is that these equations are well posed[1] for initial data with total finite energy. Moreover, since the energy is preserved by the time evolution, the solution we construct can be extended for infinite time. We also show our solution preserves any additional H^s regularity of the data and thus recover the well-known result[2] of Eardley and Moncrief [EM] concerning the global regularity of smooth solutions to these equations. Our methods rely in a fundamental way on the new estimates for solutions to the linear wave equation derived in [KlMa] as well as the choice of the Coulomb gauge condition, relative to which the nonlinear terms of the corresponding equations are shown to exhibit the null structure needed to apply the results of [KlMa]. Once all the desired results are established relative to the Coulomb gauge, we can also construct solutions in the more familiar temporal gauge.

The Maxwell-Klein-Gordon equations are the Euler-Lagrange equations associated to the Lagrangian

$$L = -\frac{1}{4}F_{\alpha\beta}F^{\alpha\beta} - \frac{1}{2}D_\mu\phi\overline{D^\mu\phi}$$

where $F_{\alpha\beta}$ is the electromagnetic field and ϕ is the complex scalar field with which F is coupled. Relative to a real potential A_α, we have

$$F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha \tag{1.1.a}$$

$$D_\mu\phi = \partial_\mu\phi + \sqrt{-1}A_\mu\phi. \tag{1.1.b}$$

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[1] This means, in the traditional terminology, that we can prove both local existence and uniqueness for finite energy initial data.

[2] The results of Eardley-Moncrief apply for the more general case of the Yang-Mills-Higgs equations. We believe however that the methods we present here can in fact be extended to the general case, we hope to address this in a separate paper.