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SHARP BOUNDS ON THE NUMBER OF SCATTERING POLES IN EVEN-DIMENSIONAL SPACES

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1. Introduction and statement of results. Recently, in [8], [13], upper bounds on the number of scattering poles for a large class of compactly supported perturbations of the Laplacian in odd-dimensional spaces have been proved. On the other hand, in the case of even space dimension only few results on the problem of obtaining upper bounds on the number of scattering poles are known essentially due to Intissar (see [1], [2]). Recall that the scattering poles (known also as resonances) can be defined as poles of the meromorphic continuation of the cutoff resolvent of the corresponding operator in the cases when such a continuaton exists (see [8], [13]). When the space dimension is odd ($\geqslant 3$), the cutoff resolvent extends meromorphically on the entire complex plane $\mathbf{C}$, while in the even-dimensional case a meromorphic extension is possible only on the Riemann logarithmic surface $\Lambda = \{z\colon -\infty < \arg z < +\infty\}$. This fact makes the problem of estimating the number of scattering poles in the case of even dimension much more complicated compared with the odd-dimensional case. In particular, one could hardly use Jensen's inequality, as for example in [5], [6], [12], [13], [15].

In [1] Intissar considers counting function

$$N(r) = \#\{\lambda_j\colon r^{-\varepsilon} \leqslant |\lambda_j| \leqslant r^{\varepsilon}, |\arg \lambda_j| \leqslant \varepsilon \log r\}, \qquad r > 1,$$

for any $\varepsilon \in (0, 2^{-1/2})$, where $\{\lambda_j\}$ are the scattering poles, repeated according to multiplicity, associated to the Schrödinger operator $-\Delta + V(x)$ in $\mathbf{R}^n$, $V \in C_0^\infty(\mathbf{R}^n)$, $n \geqslant 4$ is even, and the bound

$$N(r) \leqslant Cr^{n+1}$$

is proved. In [2] this result is extended to more general operators of the form $-\Delta + V(x, D_x)$, where V is a first-order differential operator with coefficients of class $C_0^\infty(\mathbf{R}^n)$.

In the present work we study the following counting function for the scattering poles:

$$N(r, a) = \#\{\lambda_j \in \Lambda\colon 0 < |\lambda_j| \leqslant r, |\arg \lambda_j| \leqslant a\}, \qquad r, a > 1,$$

for a class of compactly supported perturbations of the Laplacian in $\mathbf{R}^n$, $n \geqslant 4$ being even. It follows from the analysis in [11] that given any a the poles $\{\lambda_j\}$ with

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