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COHERENT COHOMOLOGY, LIMITS OF DISCRETE SERIES, AND GALOIS CONJUGATION

DON BLASIUS, MICHAEL HARRIS, AND DINAKAR RAMAKRISHNAN

0. Introduction.

0.0. In this paper .we establish some results concerning the action of automorphisms of **C** on certain (nonholomorphic) automorphic forms on symmetric spaces. The forms in question either define coherent cohomology classes on Shimura varieties or are related to such forms via known cases of the principle of functoriality. Our first result asserts that cuspidal automorphic representations with infinity type in the *discrete series* or *nondegenerate limits of discrete series* define nontrivial coherent cohomology classes. Since the coherent cohomology groups admit rational structures invariant under the Hecke algebra, we get the rationality over a number field **T** of such automorphic forms. By a general principle, we show moreover that **T** is actually contained in a *CM* field, i.e., a totally imaginary quadratic extension of a totally real number field. Further, under a *largeness of parameter* hypothesis, we show that all classes in a given cohomology group belong to a *unique* infinity type as above. (Actually, our methods prove such a solitude result for unitary representations with $\mathcal{N}$-cohomology with sufficiently large parameter on any semisimple Lie group having the same rank as its maximal compact subgroup. This appears to be new for singular parameters, though a more precise result is known in the regular case [W2].) When applied in conjunction with the principle of functoriality, the above result implies, for arithmetic automorphic forms on $GL(4)$ admitting a suitable polarization (see §0.6 and §5), that the unramified Hecke eigenvalues generate a number field **T**. Further, for each embedding τ of **T** in **C**, our results imply under a hypothesis that there exists another such form belonging to the τ-conjugate system of eigenvalues.

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