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MULTIPLE SOLUTIONS TO THE PLATEAU PROBLEM FOR NONCONSTANT MEAN CURVATURE

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1. Introduction. The questions that we investigate originate from the classical so-called plateau problem. Γ being a Jordan curve in $\mathbf{R}^3$, the Plateau problem consists in finding disc-type surfaces of minimal area spanning Γ. Such a surface has mean curvature zero, and it may be parametrized by a function

$$u: D^2 = \{(x, y) \in \mathbf{R}^2/x^2 + y^2 < 1\} \to \mathbf{R}^3$$

which satisfies

$$\Delta u = 0 \qquad \text{in } D^2, \tag{1.1}$$

$$|u_x|^2 - |u_y|^2 = u_x . u_y = 0 \qquad \text{in } D^2, \tag{1.2}$$

$$u_{|\partial D^2} \text{ is a continuous monotone parametrization of } \Gamma. \tag{1.3}$$

Conversely, a solution to (1.1), (1.2), (1.3) parametrizes, away from branch points, a surface with mean curvature zero spanning Γ, whose area is not necessarily minimal but is stationary.

In order to give this problem a variational structure, one often prefers to consider the related Dirichlet-type problem

$$\text{(I)} \qquad \begin{cases} \Delta u = 0 & \text{in } D^2 \\ u = \gamma & \text{on } \partial D^2, \end{cases}$$

where γ is a given function from ∂D^2 to $\mathbf{R}^3$. Then one can take advantage of the freedom that we have in the choice of γ as a parametrization of Γ to get the conformality condition (1.2) satisfied; see for instance [7], [17], [18], and references therein.

Since a solution to the classical Plateau problem has mean curvature zero, a natural generalization is to seek for surfaces spanning Γ whose mean curvature is a given constant $H \in \mathbf{R}$. Equation (1.1) is then replaced by

$$\Delta u = 2H u_x \wedge u_y \qquad \text{in } D^2, \tag{1.4}$$

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