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PROPAGATION OF SINGULARITIES FROM SINGULAR AND INFINITE POINTS IN CERTAIN COMPLEX-ANALYTIC CAUCHY PROBLEMS AND AN APPLICATION TO THE POMPEIU PROBLEM

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0. Introduction. In this paper we study the local propagation of singularities of the solution to a certain class of complex-analytic Cauchy problems, namely

$$(0.1)\qquad \begin{cases} \dfrac{\partial^2 u}{\partial z \partial w} + a(z, w)\dfrac{\partial u}{\partial z} + b(z, w)\dfrac{\partial u}{\partial w} + c(z, w)u = g(z, w) \\ u = |\nabla u| = 0 \quad \text{on}\, \Gamma, \end{cases}$$

where a, b, c, and g are analytic and Γ is an irreducible analytic hypersurface in some domain in $\mathbb{C}^2$. Near a nonsingular and noncharacteristic point on Γ (see the next paragraph for the definition of characteristic points) the well-known Cauchy-Kowalevskaya theorem asserts that there is a unique analytic solution to (0.1). The analytic continuation of such a solution along a curve on Γ must develop a singularity at each irreducible characteristic point (see, e.g., [S, §9.2]). By an irreducible characteristic point we mean a point which is characteristic on some local irreducible component of Γ; the definition of characteristic points given in the next section includes all singular points and a singular point may be noncharacteristic on each local irreducible component of Γ, e.g., the origin on $\{w^2 - z^2 = 0\}$. We also know that the singularity must propagate along one of the bicharacteristics emanating from that point. If the point is nonsingular, then it follows from Leray's general theory (see [L]) that the singularity propagates along the bicharacteristic tangent to Γ. The main result in Section 1 is that, if the point is an irreducible singular point, then the singularity propagates along both bicharacteristics.

In Section 2 we consider the propagation of singularities from points at infinity of the solution to a less general complex-analytic Cauchy problem, namely $a \equiv b \equiv 0$ and c and g are constant in (0.1). We find that, under a certain hypothesis on Γ, the singularity propagates along a bicharacteristic into the finite space. The propagation of singularities of the solution to this specific Cauchy problem is related to the so-called Pompeiu problem. In Section 3 we investigate the significance of the results in Sections 1 and 2 for the two-dimensional Pompeiu problem and find that we are able to sharpen a result from [E2]. Indeed, Theorem 3.1 in combination with a result from [E1] allows us to characterize the disk as the only domain without

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