Vol. 73, No. 2 DUKE MATHEMATICAL JOURNAL © February 1994

HOLONOMIC q-DIFFERENCE SYSTEM OF THE FIRST ORDER ASSOCIATED WITH A JACKSON INTEGRAL OF SELBERG TYPE

KATSUHISA MIMACHI

Dedicated to Professor R. Askey on his 60th birthday.

0. Introduction. Holonomic q-difference systems are studied in various aspects. Remarkable progress in the theory of holonomic q-difference systems is made by the recent discovery of the links both with the q-Knizhnik-Zamolodchikov equation, by I. B. Frenkel and N. Yu. Reshetikhin [10], and with the elliptic solution of the Yang-Baxter equation, by K. Aomoto et al. [5]. Despite such progress, very few examples of holonomic q-difference systems are known [15], [16]. Hence it seems very fundamental and important, even for developing the general theory of holonomic q-difference equations [2], [6], [7], [19], to construct a holonomic q-difference system with an explicit coefficient.

The aim of this article is the construction of a matrix-valued holonomic q-difference system, which is associated with the so-called *Jackson integral of Selberg type*. See [11] and [12] for the q-Selberg integrals.

One of the Jackson integrals of Selberg type that is discussed in this article is defined by

$$\int \prod_{i=1}^{n} t_i^{\alpha_i} \frac{(q^{\beta_0'}t_i/t_0;\, q)_\infty (q^{\beta_{n+1}'}t_{n+1}/t_i;\, q)_\infty}{(q^{\beta_0}t_i/t_0;\, q)_\infty (q^{\beta_{n+1}}t_{n+1}/t_i;\, q)_\infty} \prod_{1 \leqslant i<j \leqslant n} \frac{(q^{\gamma'}t_j/t_i;\, q)_\infty}{(q^{\gamma}t_j/t_i;\, q)_\infty}\, d\tau,$$

where $d\tau = d_q t_1 \wedge \cdots \wedge d_q t_n$, $t_0 = 1$, and $t_{n+1} = z$, and the integral is considered to be taken over the suitable cycle. In this article, to get the *symmetry* (precise meaning given below) among the variables $t_1, \ldots, t_n$ of integral, we impose the conditions $\alpha_k - \alpha_{k+1} = \gamma - \gamma'$ $(1 \leqslant k \leqslant n-1)$ and $\gamma + \gamma' = 0$ upon its exponents. It is noted that they are different from the symmetry condition in [4].

Associated with the above integral, we construct the holonomic q-difference system of rank $n+1$. We construct the system with the help of the basis of the q-twisted de Rham cohomology associated with the integral [3]. The rank of the q-twisted de Rham cohomology $H_\Phi^n(V, d_q)$ associated with our integral is $2(n+2)^{n-1}$, which is greater than the rank $(n+1)!$ of the classical $(q=1)$ case. The cohomology $H_\Phi^n(V, d_q)$ is of course closed under the action of the q-shift operator T_z: $(T_z f)(z) = f(qz)$. Moreover, for our purpose, we seek an $(n+1)$-dimensional T_z-submodule in this space. Such construction is similar to the classical case given

Received 29 March 1993. Revision received 9 September 1993.