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CRYSTAL BASES OF MODIFIED QUANTIZED ENVELOPING ALGEBRA

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0. Introduction.

0.1. G. Lusztig gives the crystal base on the modified quantized enveloping algebra $\tilde{U}_q(\mathfrak{g})$ in [L2]. The algebra $\tilde{U}_q(\mathfrak{g})$ is obtained from the quantized universal enveloping algebra $U_q(\mathfrak{g})$ by modifying the torus part $\bigoplus_{h \in P^*} \mathbf{Q}(q)q^h$ to $\bigoplus_\lambda \mathbf{Q}(q)a_\lambda$, where a_λ is the projector to the weight space of weight λ (see §1.2). He gives also several conjectures on its properties in [L3]. The purpose of this paper is to study the structure of crystal bases $B(\tilde{U}_q(\mathfrak{g}))$ of $\tilde{U}_q(\mathfrak{g})$ and to give an affirmative answer to some of his conjectures.

0.2. Let us explain the results obtained here more precisely. We establish that the crystal structure of $\tilde{U}_q(\mathfrak{g})$ is described by those of $U_q^-(\mathfrak{g})$ and $U_q^+(\mathfrak{g})$. Namely, let $B(\infty)$ be the crystal base of $U_q^-(\mathfrak{g})$ and $B(-\infty)$ the one of $U_q^+(\mathfrak{g})$. Let T_λ be the crystal consisting of a single element of weight λ. Then the crystal base of $\tilde{U}_q(\mathfrak{g})$ is isomorphic to the direct sum of $B(\infty) \otimes T_\lambda \otimes B(-\infty)$ (Theorem 3.1.1). This fact is a reflection of $\tilde{U}_q(\mathfrak{g}) = \bigoplus_\lambda U_q^-(\mathfrak{g}) \otimes U_q^+(\mathfrak{g}) \otimes \mathbf{Q}(q)a_\lambda$. The algebra $\tilde{U}_q(\mathfrak{g})$ has the antiautomorphism $*$ that sends e_i, f_i to themselves and a_λ to $a_{-\lambda}$. We prove that the crystal base is stable by $*$ (Theorem 4.3.2). This is one of the conjectures of Lusztig [L3]. This automorphism sends $b_1 \otimes t_\lambda \otimes b_2 \in B(\infty) \otimes T_\lambda \otimes B(-\infty) \subset B(\tilde{U}_q(\mathfrak{g}))$ to $b_1^* \otimes t_{-\lambda - wt b_1 - wt b_2} \otimes b_2^*$. By this automorphism, $B(\tilde{U}_q(\mathfrak{g}))$ has another crystal structure. These two structures are compatible (see §5), and $B(\tilde{U}_q(\mathfrak{g}))$ may be regarded as a crystal over $\mathfrak{g} \oplus \mathfrak{g}$. This is a reflection of the $U_q(\mathfrak{g})$-bimodule structure of $\tilde{U}_q(\mathfrak{g})$.

0.3. In [K2], the author introduces "the dual algebra" $A_q(\mathfrak{g})$ of $U_q(\mathfrak{g})$ and its crystal base $B(A_q(\mathfrak{g}))$. This algebra has the Peter-Weyl–type decomposition

$$A_q(\mathfrak{g}) = \bigoplus_{\lambda \in P_+} V^r(\lambda) \otimes V(\lambda).$$

Here P_+ is the set of dominant integral weights and $V(\lambda)$ and $V^r(\lambda)$ are the left and right highest-weight module with highest weight λ. Accordingly $B(A_q(\mathfrak{g}))$ has the crystal structure

$$B(A_q(\mathfrak{g})) = \bigoplus_{\lambda \in P_+} B(\lambda) \otimes B(\lambda).$$

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