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# ON POISSON PAIRS ASSOCIATED TO MODIFIED R-MATRICES

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**1. Introduction.** Let $\mathfrak{g}$ be a semisimple Lie algebra over $\mathbf{C}$ and $G$ its adjoint group. Consider a homogeneous $G$-manifold $M = G/H$. Any element $X \in \mathfrak{g}$ defines a holomorphic vector field $\rho(X)$ on the manifold $M$ in the following way: $\rho(X)f(m) = (d/dt)(f(e^{-tX}m))|_{t=0}, f \in Fun(M)$. The correspondence $X \mapsto \rho(X)$ is a representation of $\mathfrak{g}$ into the space $Vect(M)$ of all holomorphic vector fields on $M$. Let us fix an element $R \in \bigwedge^2 \mathfrak{g}$ and associate to it the operator

$$f \otimes g \to \{f, g\}_R = \mu\langle(\rho \otimes \rho)R, df \otimes dg\rangle, \qquad f, g \in Fun(M). \tag{1}$$

Hereafter $\mu$ is the usual commutative multiplication in the space of holomorphic functons $Fun(M)$

$$\mu\colon Fun(M)^{\otimes 2} \to Fun(M)$$

and $\langle\ ,\ \rangle$ stands for the pairing between vector fields and differential forms.

Let us consider two conditions.

(i) $R$ satisfies the classical Yang-Baxter equation; i.e., $[[R, R]] = 0$ where

$$[[R, R]] = [R^{12}, R^{13}] + [R^{12}, R^{23}] + [R^{13}, R^{23}].$$

(It is clear that $[[R, R]] \in \bigwedge^3 \mathfrak{g}$ for any $R \in \bigwedge^2 \mathfrak{g}$.)

(ii) The operator (1) defines a Poisson bracket; i.e., it satisfies the Jacobi identity (since the antisymmetricity and the Leibnitz identity are fulfilled automatically).

It is obvious that the implication (i) ⇒ (ii) is true. However (ii) could be fulfilled even if the condition (i) fails. In the present paper we investigate the following.

PROBLEM. *Let $\mathfrak{g}$ be a simple Lie algebra. Describe all orbits $\mathcal{O}$ in $\mathfrak{g}^*$, such that the condition (ii) is fulfilled, where $R$ is a modified $R$-matrix.*

These orbits are said to be the *orbits of R-matrix type*.

In fact the problem under consideration may be formulated without any $R$-matrix, and therefore the property of an orbit to be of the $R$-matrix type does not depend on a particular choice of $R$-matrix. More exactly, there exists a unique (up to a scalar multiple) $G$-invariant 3-form on $\mathfrak{g}^*$ and an orbit $\mathcal{O}$ is of $R$-matrix type if and only if the restriction of this 3-form on $\mathcal{O}$ is identically zero (cf. Section 2).

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