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CERTAIN DIRICHLET SERIES ATTACHED TO AUTOMORPHIC FORMS OVER IMAGINARY QUADRATIC FIELDS

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Introduction. Let $\mathbb{S} = \mathbb{C} + \mathbb{R}^+ j$ be the hyperbolic 3-space embedded in the ring of Hamilton quaternions. $\mathbb{S}$ can be identified with the quotient space $SL_2(\mathbb{C})/SU_2$ through the group action defined by

$$\alpha(z) = (az + b)(cz + d)^{-1} \qquad \text{if } \alpha = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL_2(\mathbb{C}),$$

and $\alpha(z) = (\sqrt{\det(\alpha)^{-1}}\alpha)(z)$ if $\alpha \in GL_2(\mathbb{C})$. Let $\mathscr{L}$ be the Laplace-Beltrami operator on $\mathbb{S}$, and let F be an imaginary quadratic number field. The automorphic forms to be considered here are eigenfunctions of $\mathscr{L}$ which are invariant under a subgroup of $GL_2(F)$ and satisfy a certain growth condition. Such an automorphic form has the Fourier expansion

$$f(u + vj) = c(0, v) + \sum_{0 \neq \xi \in L} \theta(\xi) v K(4\pi |\xi| v) e^{2\pi i(\xi u + \bar{\xi}\bar{u})} \qquad (u \in \mathbb{C}, v > 0)$$

where L is a two-dimensional lattice in F and K is a Bessel function. The Dirichlet series ordinarily associated with f is $\mathscr{D}(s, f) = \sum_{0 \neq \xi \in L} \theta(\xi) |\xi|^{-s}$. This type of Dirichlet series has been investigated extensively. We already have some pleasant results in the general theory concerning this type of Dirichlet series, such as analytic continuations and functional equations. (See [JL] and [We].) In this paper, we consider a kind of subseries of $\mathscr{D}(s, f)$, namely,

$$\mathscr{R}(s, f) = \sum_{0 \neq \xi \in L \cap \mathbb{Q}} \theta(\xi) |\xi|^{-s}.$$

We shall prove that $\mathscr{R}(s, f)$ has an analytic continuation and satisfies a functional equation if f is a cusp form (Theorem 2.3 and Theorem 2.7). The method is based on a pullback from $GL_2(\mathbb{C})$ to $GL_2^+(\mathbb{R})$, through which an automorphic form on $\mathbb{S}$ is transformed to a real analytic function on the complex upper half-plane which is invariant under certain subgroup of $GL_2^+(\mathbb{Q})$ and satisfies a growth condition. Eisenstein series on the complex upper half-plane play a basic role in our method.

To get an Euler product, we adopt the adèlic viewpoint from Section 3 on. We consider automorphic forms ϕ on $G_{\mathbf{A}}$, the adèlization of $GL_2(F)$. By establishing a

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