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A CRITICAL GROWTH RATE FOR THE PLURICOMPLEX GREEN FUNCTION

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Introduction. For each bounded convex domain G in $\mathbb{C}^N$ which contains the origin, the pluricomplex Green function g with pole at the origin is the largest negative plurisubharmonic function on G for which $g - \log|z|$ is bounded from above. It was introduced by Lempert [7] while looking for a several variables substitute for the Riemann conformal mapping of G onto the disc. In the present paper we relate the boundary behavior of g with the following functional analysis problem:

Let $A(G)$ denote the Fréchet space of all analytic functions on G. If $P(z) = \sum_{\alpha \in \mathbb{N}_0^N} a_\alpha z^\alpha$ is a nonzero entire function on $\mathbb{C}^N$ of at most order one and zero type, Martineau [9] proved that

$$P(D)f := \sum_{\alpha \in \mathbb{N}_0^N} a_\alpha f^{(\alpha)}$$

defines a surjective continuous linear map $P(D)$: $A(G) \to A(G)$. We consider the problem whether there is a continuous linear map R: $A(G) \to A(G)$ which assigns to each $h \in A(G)$ a solution of the equation $P(D)R(h) = h$. Such a map is called a "solution operator" of $P(D)$: $A(G) \to A(G)$.

In the 1960s Trèves [25] proved that each partial differential operator (of finite order) admits a solution operator on $A(\mathbb{C}^N)$. This result was extended in the 1980s to partial differential operators of infinite order. For $N = 1$ this has been done by Taylor [24] and Schwerdtfeger [21], for $N > 1$ by Meise and Taylor [11]. An extension of these results to convex Reinhardt domains G of $\mathbb{C}^N$ is contained in [13]. In [14] we characterized for an arbitrary convex domain G of $\mathbb{C}$ when $P(D)$ admits a solution operator on $A(G)$.

To formulate our result for a given bounded convex domain G of $\mathbb{C}^N$ which contains the origin, let $G_x := \{z \in G | g(z) < x\}$, $x < 0$, denote the level sets of the pluricomplex Green function g. If H is the support function of the convex set G, we consider, moreover, a certain extremal plurisubharmonic function v_H, which was investigated in [17]. v_H is the largest plurisubharmonic function on $\mathbb{C}^N$ with $v_H \leqslant H$ and such that $v_H - \log(1 + |z|)$ is bounded from above. Functions like v_H have been introduced by Siciak [22].

THEOREM. *For each bounded convex domain G of $\mathbb{C}^N$ which contains the origin, the following assertions are equivalent:*

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