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UNITARY NILPOTENT GROUPS AND THE STABILITY OF PSEUDOISOTOPIES

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Two homeomorphisms from a topological space onto itself are *isotopic* if they are homotopic through homeomorphisms. This defines an equivalence relation whose equivalence classes are called *isotopy classes.* A useful step in analyzing these classes is the introduction of a coarser equivalence relation called a *pseudoisotopy*; specifically, two homeomorphisms h_0 and h_1 from a space X to itself are *pseudoisotopic* if there is a homeomorphism H from $X \times [0, 1]$ onto itself such that it sends $X \times \{0\}$ and $X \times \{1\}$ onto themselves by h_0 and h_1 respectively. The difference between an isotopy and a pseudoisotopy for a manifold M is measured by the 0-dimensional homotopy group of a geometrically defined *concordance space* $C(M)$; frequently this object is also called the pseudoisotopy space of M. Such spaces exist over the topological, *PL*, and smooth categories and have been studied extensively over the past twenty years (specifically, in work of Cerf [Ce], Burghelea and Lashof [BL], Hatcher and Wagoner [HW], Hatcher [H2], Waldhausen [Wd1], and Igusa [Ig0–2]). In particular the work of Hatcher, Wagoner, and Igusa leads to a complete description of $\pi_0(C(M))$ in terms of higher algebraic K-theory invariants if $\dim M \geqslant 5$. The main results of this paper state that the Hatcher-Wagoner invariants do *not* necessarily detect all elements of $\pi_0(C(M))$ if $\dim M = 3$ (see Theorems 1 and 3) and that certain nonzero invariants are realized by elements of $\pi_0(C(N))$ for certain 3-manifolds N^3 (see Theorems 2 and 3).

One of the most important features of the pseudoisotopy spaces is the *stability property*. Namely, let $\Sigma\colon C(M) \to C(M \times I)$ be the suspension map given by $\Sigma(f) = f \times id_I$. Then the stability of pseudoisotopies asserts that "*the map* $f \to f \times id_I$ *is k-connected provided* $\dim M \gg k$" (see [H1], [Ig2]). This implies that the space of stable pseudoisotopies $\mathscr{C}^{\text{stab}}(M) = \lim_{\overrightarrow{n}} C(M \times I^n)$ is a homotopy functor which in turn enables one to use a variety of additional techniques to study $\mathscr{C}^{\text{stab}}(M)$ and thus to obtain new information about $C(M)$ itself (cf. [H2], [Wd1]). On the π_0-level the stability of pseudoisotopies simply says that the iterated suspension

$$\Sigma^n\colon C(M) \to C(M \times I^n)$$

induces an isomorphism

$$\pi_0(C(M)) \overset{\approx}{\to} \pi_0(C(M \times I^n))$$

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