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GLOBAL 2-FORMS ON REGULAR 3-FOLDS OF GENERAL TYPE

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Section 1. Let V be a smooth projective variety of dimension three over $\mathbb{C}$. We would like to understand the relation between globally defined 2-forms and the pluricanonical systems on V when V is of general type; i.e., high multiples of the canonical line bundle define birational maps. The study is motivated by the following conjecture:

1.1. For a 3-fold V of general type, there is a universal integer N such that ϕ_{NK_V} defines a birational map.

It is well known that this statement holds in dimensions 1 and 2. The conjecture is true for a 3-fold of general type whose minimal model has, at worst, Gorenstein singularities. The complexity arises when the minimal model possesses singularities of index bigger than 1, which is exactly the new feature of minimal models in dimension three. One possible approach toward solving the conjecture is to study the Riemann-Roch formula provided by Reid and Fletcher [R]. The question then becomes completely combinatoric, but unfortunately contributions from the singularities to the formula seem too hard to handle. Nevertheless, with the help of the formula, we are able to obtain a bound n_0 as a function of $\chi(V, \mathcal{O}_V)$ such that $P_n(V) \geqslant 1$ for $n \geqslant n_0$ [F, Section 5].

For irregular 3-folds of general type, the conjecture is proved by virtue of the work of Kollár [K] through studying the classical Albanese map. Indeed, one may take N to be 143.

Assume $h^1(V, \mathcal{O}_V) = 0$. We call such a variety a regular 3-fold. If V is a regular 3-fold of general type, we would like to understand certain maps defined by globally defined 2-forms on V. Our study is related to the conjecture in the following simple way. Obviously, a regular 3-fold with $h^2(V, \mathcal{O}_V) \leqslant 3$ has the property that $\chi(V, \mathcal{O}_V) \leqslant 4$. If $\chi(V, \mathcal{O}_V)$ is bounded, it is conceivable that one could find such an N as a function of χ by looking at Reid's formula carefully. Naively, curves and surfaces which are images of a regular 3-fold do not provide much information as far as the study of pluricanonical systems is concerned. Suppose we could construct at least 2 linearly independent sections in $|sK_V|$ for 3-folds with $h^2(V, \mathcal{O}_V) \geqslant 4$, using special properties of maps defined by global 2-forms; then a result in [K, Corollary 4.8] says that one may take $N = 11s + 5$ for such varieties.

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