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COHOMOLOGY OF CRYSTALLINE REPRESENTATIONS

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1. Introduction. In studying the arithmetic geometry of a scheme X defined over a number field K, it is fundamental to consider the $\mathrm{Gal}(\overline{K}/K)$-module structure of the étale cohomology groups $H^i(X \otimes \overline{K}, \cdot)$ with diverse coefficients. In particular, it is sometimes unavoidable to consider, say, $H^i(X \otimes \overline{K}, \mathbf{Z}/p^n)$ for a *fixed* p, as a global Galois-module (e.g. [16]). In this case, although the action of the decomposition groups D_v for v not dividing p is relatively well understood, analyzing the action of D_v for primes v *dividing* p poses difficult problems, which must be dealt with on a quite different footing.

Consider the representations of D_v's on p-adic vector spaces: if $v|p$, call them p-adic; otherwise call them l-adic. The theory of p-adic representations has been developed extensively in recent years by Fontaine, Messing, Faltings, and many others. By associating to a p-adic representation a crystal—a certain filtered module with a Frobenius, Fontaine [7] has distinguished a full subcategory of p-adic representations, *crystalline* representations. They are the p-adic analogue of unramified l-adic representatons. In fact, the representations arising from the p-adic étale cohomology of proper and smooth schemes over K with good reduction at v, known to be unramified for v not dividing p, were shown ([11], [4]) to be crystalline for $v|p$. Moreover, in that case, the theory of p-adic periods (a nice overview of which the reader can find in [9], [14]) yields that the associated crystals are canonically isomorphic to the de Rham cohomology of the scheme, endowed with its Hodge filtration and crystalline Frobenius—the Frobenius coming from the one acting on the crystalline cohomology of the special fiber of a proper, smooth model of the scheme over $\mathrm{Spec}(\mathcal{O}_v)$, where $\mathcal{O}_v$ is the ring of integers of the completion of K at v, via the canonical isomorphism between de Rham and crystalline cohomology. There is also a theory treating p-torsion and, most interestingly, certain types of local systems.

Let now K be a number field, S a set of primes of K containing all primes above ∞ and p, $G_S = \mathrm{Gal}(K_S/K)$, where K_S is the maximal extension of K unramified outside of S, and $\mathrm{Rep}(G_S)$ the category of p-adic representatons of G_S: finite-dimensional $\mathbf{Q}_p$-vector spaces endowed with a linear and continuous action of G_S.

Consider the full subcategory $\mathrm{Rep}_{cr}(G_S)$ of $\mathrm{Rep}(G_S)$ consisting of representations crystalline at p, i.e., such that the action of a decompositon group D_v at v is crystalline if $v|p$ (see [7]). In [3] the groups of one-term extensions of $\mathbf{Q}_p$ by representations from $\mathrm{Rep}_{cr}(G_S)$ were introduced and studied in an attempt to extract the "geometric" part of Galois cohomology, and in relation to special values of L-functions.

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