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DEGENERATE FOURIER INTEGRAL OPERATORS IN THE PLANE

ANDREAS SEEGER

1. Introduction. Let X and Y be bounded open sets in $\mathbb{R}^2$ and let $\mathcal{M}$ be a hypersurface in $X \times Y$ with conormal bundle $N^*\mathcal{M}$. Let $\Lambda = N^*\mathcal{M} \backslash$zero section in $T^*X \times T^*Y$. We shall always assume that

$$\Lambda \subset T^*X\backslash 0 \times T^*Y\backslash 0 \tag{1.1}$$

where now 0 refers to the zero sections in T^*X and T^*Y respectively. If $\mathcal{M}$ is given by

$$\{(x, y); \Phi(x, y) = 0\}, \tag{1.2}$$

then (1.1) means that both Φ'_x and Φ'_y do not vanish on $\mathcal{M}$. In particular, for $(x, y) \in \mathcal{M}$ the varieties

$$\mathcal{M}_x = \{y \in Y; (x, y) \in \mathcal{M}\},$$

$$\mathcal{M}^y = \{x \in X; (x, y) \in \mathcal{M}\},$$

are smooth immersed curves in Y and X, respectively. Let $\chi \in C^\infty(X \times Y)$ be compactly supported in $X \times Y$. We consider averaging operators of the form

$$\mathcal{R}f(x) = \int_{\mathcal{M}_x} \chi(x, y) f(y)\, d\sigma_x(y) \tag{1.3}$$

where $d\sigma_x$ is a smooth density on $\mathcal{M}_x$ depending smoothly on x.

We shall examine the $L^p \to L^q$ and $L^p \to L^p_\alpha$ mapping properties of $\mathcal{R}$. (Here L^p_α denotes the standard L^p-Sobolev space.) The operators $\mathcal{R}$ are Fourier integral operators and belong to the class $I^{-1/2}(X, Y, \Lambda)$ (see [9]), and those mapping properties are well understood if Λ is locally the graph of a canonical transformation. The latter condition means that the projections $\pi_R \colon \Lambda \to T^*Y$ and $\pi_L \colon \Lambda \to T^*X$ are locally diffeomorphisms. Another way of expressing this is to say that the Monge-Ampere determinant

$$\mathcal{J}(x, y) = \det \begin{pmatrix} \Phi''_{xy} & \Phi'_x \\ {}^t\Phi'_y & 0 \end{pmatrix} \tag{1.4}$$

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